Significance of the probability of failure in slope engineering

H A D Kirsten* (Fellow)

Synopsis

A qualitative and empirical scale of the significance of the probability of failure in slopes, based on published case studies, is presented. The suitability of the scale is illustrated in terms of examples of soil, mine tailings and rock slopes. It is further illustrated that the peak strength parameters obtained in the usual way for soil and rock slopes are sufficiently reliable to calculate the probability of failure.

Significance of failure

In the absence of water impoundment effects, approximately 0.5 per cent of modern dams failed catastrophically during their active life of 50 to 100 years. Carlier established that the world-wide annual risk of catastrophic failure among an estimated 10 000 dams over 15 m high was approximately 0.01 per cent.

Tang, Yuceman and Ang developed a sophisticated model from an extensive literature survey of the ranges of variation of every factor introducing uncertainty in the design of soil slopes. On the basis of their model they calculated that the risk implicit in the design of earth slopes using factors of safety of 1.3 and 1.5 varied between 0.006 and 1 per cent for short term stability.

The theoretical findings confirmed the results on actual failure reported by Meyerhof, Feld and Carlier. The categories for less than 0.5 and 0.5 to 1.5 per cent probability of failure in Table 1 have been based on this information. Moss and Steffen advocated a maximum allowable probability of failure of 0.3 per cent for permanent haul roads in open-pit mines, which evidently fits in with the lowest class in Table 1.

Steffen recommended some risk levels of up to 30 per cent in the design of rock slopes in open-pit mines and road and railway cuttings. He distinguished between risks of failure with and without monitoring. These have largely been incorporated in Table 1. The category 50-100 per cent probability of failure is based on the finding of Moss and Steffen that a 50 per cent probability of failure corresponds on average to a factor of safety of 1.0. The class interval 20-50 per cent is rather wide, but because of the lack of supporting statistics it cannot be subdivided into more categories. Selection of a probability of failure in this range should be done with particular caution and careful judgement against the background of the particular problem. The actual values of the class interval boundaries in the table were selected in principle on the basis of a logarithmic dependence of the probability of failure on the central factor of safety or the safety margin as defined by Harr, p 407.

The proposed scale in Table 1 is based on the assumption after Lumb and Schultz that all the input parameters which are required to calculate the probability of failure follow normal distributions. Harr, p 396, has indicated that although all the parameters are beta distributed it would appear to be reasonably justified to approximate the central portions of their distributions as normal variates. Considerable discrepancies are invoked in the region of the tails of the distributions by assuming these to be normal. The most serious effects of this assumption are with regard to:

1. The region of the lower tails of the distribution of friction angle in general and of cohesion for clay.
2. The larger proportion of the distribution of cohesion for predominantly frictional materials.

In the case of low probabilities of failure, of the order of 5 per cent and less, these effects have a significant influence on the accuracy of the calculation. For such low probabilities of failure an incorrect probability

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density function of the strength parameters could in fact change the probability of failure to a very large extent. In the case of larger probabilities of failure when the central portions of the strength distributions play a greater role, the accuracy of the calculation is relatively insensitive to the type of probability density function for the strength parameters. This is confirmed by Harr\(^1\), p 420, and by Förster and Weber\(^2\). The effect of the type of the probability density function of the strength parameters on small values of probability of failure in the proposed significance scale has not been established, but it is not considered to impose a serious limitation on the general use of the scale.

The risk of failure may be defined as the product of the probability of failure and the probability of occurrence of a block or zone of unstable ground. The data in Table 1 refers only to the probability of failure, or to the risk of failure when the probability of occurrence of unstable ground is equal to unity. In other words the data in the table can only be applied to plane problems or to situations in which the configuration of the unstable zone is known. The concept risk of failure is dealt with more fully by for example Shuk\(^1\) and Moss and Robertson\(^1\).

The criteria given in Table 1 correspond to the maximum allowable probabilities of failure as they would correspond to limited test data usually available. If extensive testing is available and the designer is extensively experienced, the risk levels given in respect of the various qualitative design criteria would be considerably reduced. Yuceman and Ang\(^1\) deal with this aspect at considerable length.

The qualitative descriptions in Table 1 in respect of the design situation were selected empirically in terms of the permanence of the slope, the public liability and the type and extent of the monitoring involved in the particular case. The descriptions in the natural situation were selected on an empirical basis in terms of the frequency of evident slope failures in the surrounding natural environment and in terms of the qualitative rate of unstable creep movements.

### Back analysis of cohesion and friction angle from probability of failure

This is the first example in terms of which the instability of the proposed significance scale can be assessed. It comprises a soil slope in which a circular arc failure is considered in a deeply weathered dolerite sill. Shortly after a cut was made in a natural slope considerable movement of the face occurred. This was subsequently accompanied by extensive ravelling of the face. The cut was approximately 50 m deep and inclined at 53°. The natural slope was inclined at approximately 26°.

The stability of a relatively shallow slope is most sensitive to the cohesion. A small amount of cohesion or a small change in cohesion on a potential plane of failure has a very significant effect on the stability of such a slope. In such cases the probability of failure of the slope is significantly influenced by both the average as well as the variance of the cohesion distribution.

The cohesion parameter is highly sensitive to sample disturbance and experimental bias in the present example, the results of laboratory direct and triaxial shear tests on several undisturbed samples are compared with the cohesion and friction angle distributions obtained from back analysis of the probability of slope failure.

**Results of direct shear tests**

Seven undisturbed samples were tested in the shear box apparatus at natural moisture content. Both peak and residual shear strengths were measured at normal stresses up to 500 kPa. The results are summarized in Table 2. Although the tests were carried out at a rapid loading rate, the parameters obtained were taken as effective because of the relatively high permeability of the soil.

### Table 2: Results of direct shear tests

<table>
<thead>
<tr>
<th>Cohesion (kPa)</th>
<th>Friction Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peak</td>
<td>Residual</td>
</tr>
<tr>
<td>Average</td>
<td>37</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>13.32</td>
</tr>
</tbody>
</table>

**Results of triaxial tests**

Three specimens were prepared from undisturbed block samples and tested in the triaxial apparatus. The tests were drained and were carried out at natural moisture content at cell pressures up to 400 kPa. All three specimens failed on inclined planes. The average values determined from a straight failure envelope for the cohesion and friction angle were

### Table 1: Comparative significance of probability of failure

<table>
<thead>
<tr>
<th>Probability of failure (%)</th>
<th>Design criteria on basis of which probability of failure is established</th>
<th>Aspects of natural situation in terms of which probability of failure can be assessed</th>
</tr>
</thead>
<tbody>
<tr>
<td>50-100</td>
<td>Effectively zero</td>
<td>Frequency of evidentiary slope failures</td>
</tr>
<tr>
<td></td>
<td>Public access forbidden</td>
<td>Slope failures generally evident</td>
</tr>
<tr>
<td>20-50</td>
<td>Very very short term (temporary open-pit mines) — untenable risk of failure in temporary civil slope</td>
<td>Clear evidence of creeping valley sides</td>
</tr>
<tr>
<td></td>
<td>Public access forcibly prevented</td>
<td>Significant number of unstable slopes works</td>
</tr>
<tr>
<td>10-20</td>
<td>Very short term (quasi-temporary slopes in open-pit mines — undesirable risk of failure in quasi-temporary civil works)</td>
<td>Continuous monitoring with sophisticated instruments</td>
</tr>
<tr>
<td></td>
<td>Public access actively prevented</td>
<td>Some unstable slopes evident</td>
</tr>
<tr>
<td>5-10</td>
<td>Short term (semi-temporary slopes in open-pit mines, quarries or civil works)</td>
<td>Continuous monitoring with simple rudimentary instruments</td>
</tr>
<tr>
<td></td>
<td>Public access prevented</td>
<td>Odd unstable slope evident</td>
</tr>
<tr>
<td>1.5-5</td>
<td>Medium term (semi-permanent slopes)</td>
<td>Continuous monitoring</td>
</tr>
<tr>
<td></td>
<td>Public access discouraged</td>
<td>No ready evidence of unstable slopes</td>
</tr>
<tr>
<td>0.5-1.5</td>
<td>Long term (quasi-permanent slopes)</td>
<td>Extremely slowly creeping valley sides</td>
</tr>
<tr>
<td>Less than 0.5</td>
<td>Very long term (permanent slopes)</td>
<td>Incidental superficial monitoring</td>
</tr>
</tbody>
</table>

**Notes:**
1. The rate of movement implied in the natural situation is with regard to geological time. The qualitative assessments are further given with regard to a significant number of locations\(^1\) in which failure can occur.
2. The lateral extent of a location where failure can occur is of the order of the height of the affected slope.
3. Small open-pit mines lie in the range 5 to 15\(^1\) and per cent allowable probability of failure depending upon the extent of monitoring as given. The corresponding range for large open-pit mines is 15 to 30\(^1\) per cent.
9.4 kPa and 21.9° respectively. It is evident that these values are greatly different from those determined in the direct shear test (Table 2).

**Determination of probability characterization curves**

The natural and excavated slopes considered in this problem are shown in cross-section in Figs 1 and 2. In both cases the zones in which the failure surface would most likely occur were identified by deterministic limit equilibrium analysis. A range of representative input parameters was used for this purpose.

Ten potential loci for the failure surface were selected arbitrarily from these zones as shown in Figs 1 and 2. The following procedure was followed in respect of each of the ten surfaces. A normal distribution of friction angles at an average of 25° and a standard deviation of 2.5° was selected arbitrarily. A normal distribution of cohesion of a certain average and standard deviation was selected. Likewise a normal distribution of the porewater pressure parameter, \( \psi \), was selected at an average of 0.1 and a standard deviation 0.05. Two hundred values were selected randomly from each of the chosen friction angle, cohesion and pore pressure parameter distributions. For each of the two hundred sets of values for the parameters a factor of safety was determined according to the Janbu\(^{8}\) simplified method.

This procedure provided a total of 2,000 factors of safety for the ten potential loci of the failure surface, from which the probability of failure was determined as the ratio of the number of factors of safety less than unity to the total number of cases analysed. This determination of the probability of failure on the basis of the histogram of values of factor of safety was selected to avoid the problem of inaccuracy of the tails of mathematically fitted probability density functions. In this way the probability of failure was determined for a range of averages and standard deviations of the cohesion distribution. The entire procedure was furthermore repeated for friction angle distributions of averages of 29° and 34° and standard deviations of 2.5° in both cases.

The results were recorded as in Fig 3, in which the probability of failure is plotted against the lower 95 per cent confidence limit of cohesion. The various curves in every graph represent lines of constant population ranges of cohesion.

**Interpretation of probability characterisation curves**

From the general appearance of the natural slopes in the region of the cutting, of which some were subject to slow creep movements and some were essentially stable, the probability of failure in the natural slope was assessed in terms of Table 1 to be of the order of 1.5 per cent.

The displacements of the cut face were influenced very significantly by heavy rains. The magnitude of the displacements after the first rainy season amounted to well over 100 mm. Extensive cracks along joints across the face, tens of metres in length, also developed and were 25 mm wide in places. Although no major tension cracks could be observed along the crest of the slope it was considered, in view of the magnitude of the displacements, that such cracks could well be disseminated readily in the strongly jointed dolerite soil. In terms of these observations the excavated face was considered to be subject to a probability of failure in the upper range of the category 20 to 50 per cent as compared with the value of 1.5 per cent for the natural slope.

A reasonable estimate for the lower 95 per cent confidence limit of cohesion was assumed to be 5 kPa. For this figure and for probabilities of failure of 1.5 per cent for the natural slope and 45 per cent for the excavated face the relevant population ranges of cohesion may be read from Fig 3. The results are plotted in Fig 4. The intersection of the two curves defines the required actual population range and average of cohesion and friction angle respectively. The detailed results are summarized in Table 3 together with the figures obtained from the laboratory tests.

The values for the peak strength parameters obtained from the direct shear tests on undisturbed samples and from the back analysis of the probability of failure are in good agreement. If a value of 10 kPa is taken for the lower 95 per cent confidence limit of cohesion a slightly different result will be obtained. The average of cohesion will not change but the average of the friction angle will decrease by an insignificant amount of one degree. The estimates of the probabilities of failure for the natural and excavated slopes are somewhat subjective.

The average total shearing strength which results from the back analysed parameters is, however, not significantly sensitive to the particular estimates of the relevant probabilities of failure. For a probability of failure of 45 per cent for the excavated slope, for example, a range in probability of failure from 0.5 to 3.5 per cent for the natural slope results in a variation in the average total shearing strength of 2.6 per cent. This is despite the fact that the average cohesion and friction angle parameters vary very considerably. For a probability of failure of 70 per cent for the excavated slope, a range in probability of failure from 0.5 to 3.5 per cent for the natural slope results in a variation in the average total shearing strength of 3.9 per cent. In this instance the average cohesion and friction angle parameters do not vary significantly.

It follows from this discussion that the cohesion and friction angle parameters are related. This is the case because the average total shearing strength is non-linearly related to the first invariant of stress. This phenomenon underlies all the examples given in this presentation. Instead of sampling randomly from the individual cohesion and friction angle distributions in generating the population density distributions for the factor of safety, the average total shearing strength distribution should be used at the appropriate value of stress as discussed by Marek.

![Graph showing probability characterization curves](image-url)

**Fig 1:** Cross-section of natural slope showing zone in which failure surface will most likely be located

**Fig 2:** Cross-section of excavated slope showing zone in which failure surface will most likely be located

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**Table 3: Summary of results on strength parameters**

<table>
<thead>
<tr>
<th>Method of determination</th>
<th>Cohesion (kPa)</th>
<th>Friction angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct shear test</td>
<td>Peak: 11-37-63</td>
<td>Residual: 31-36-41</td>
</tr>
<tr>
<td></td>
<td>Peak: 25-35</td>
<td>Residual: 21-26-31</td>
</tr>
<tr>
<td>Triaxial shear test</td>
<td>88</td>
<td>22</td>
</tr>
<tr>
<td>Back calculation from probability of failure</td>
<td>5-35-42</td>
<td>30-35-38</td>
</tr>
</tbody>
</table>

**Note:** Bar indicates average and extreme figures the upper and lower 95 per cent confidence limits.
and Savely20. However, the computer programme21 which was used in all the examples presented in the paper requires the input data to be specified in terms of the cohesion and friction angle parameters.

For predominantly frictional materials, it is clear that the shear box test provides sufficiently reliable peak strength parameters for the probability for failure of a soil slope to be evaluated by the proposed significance scale. The values for the parameters determined from the triaxial test are unreliable mainly because of the limited number of tests carried out.

**Probability of failure in mine tailings dam**

This second example is based on a major failure in a mine tailings dam reported by Fourie and McPhail22. The slope was inclined at approximately 30°, was 32 m high and was situated mainly on 5 m deep shattered and slickensided clay. It was instrumented with piezometers which showed a relatively high phreatic surface. Failure occurred in principle along a soft clay layer in the foundation and along a 63° inclined plane intersecting the top of the dam some 5 m behind the crest.

Fourie and McPhail determined population distributions for the cohesion and friction angle of the tailings and the clay from extensive shear box tests. Twenty different failure surfaces were randomly selected within the confines of the soft foundation layer and the 63° plane through the tailings deposit. One hundred sets of strength parameters were selected randomly from the given population distributions for each failure surface. For every set of strength parameters a factor of safety was determined according to the Janbu19 simplified method. The probability of failure of 48 per cent was found to be the number of factors of safety less than unity, divided by 2000, the total number of factors of safety calculated. This result confirms the upper
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In this example the failure was primarily dominated by the soft clay foundation. The authors conducted some sensitivity analyses into the effect of the fissures in the shattered and slickensided clay on the average factor of safety and the associated probability of failure. They found, in principle, that for the clay the strength was dominated by cohesion there was relatively good agreement between the remoulded total peak strength and the back analysed total shearing strength.

Significance of probability of a failure in rock slopes

The third and fourth examples respectively involve failures in rock slopes on a construction site and in an open cast mine along the crest of which an existing railway line is situated.

Construction works

The slope in this example formed the side of a cutting through a shale. The maximum depth of the cutting was 16 m and the sides were inclined at approximately 68° as shown in Fig 5. The shale was moderately weathered, highly jointed and sub-horizontally bedded. The joints were very long, of the order of several tens of metres, and filled with a firm, silty clay gouge up to 50 mm thick in places. The cutting was parallel to the strike of the bed and to the strike of the main adversely inclined joint set.

A failure, approximately 90 m in length, occurred on one side of the cutting during excavation. It occurred on a deeply-seated surface shortly after a heavy downpour of rain. The failure surface comprised segments along the bedding and along a steep adversely inclined joint. For the purposes of a back analysis a tension crack of 2 m deep was assumed. The failure surface was determined by accurate survey. The cross-section shown did not vary significantly over the length of the failed zone.

Shear box tests were carried out at a relatively slow rate of loading on remoulded samples of the gouge material at optimum moisture content. The average obtained for the effective friction angle amount to 23°. An associated standard deviation of 3.5° was arbitrarily selected.

Although the gouge was of considerable thickness the separation and waviness of the joints varied considerably, so much so that the joint walls came into light contact over a considerable portion of the area of the joint planes. This phenomenon was allowed for by determining the joint strength parameters from the Barton and Choubey23 formula:

\[ \phi = \arctan \left( \frac{\tan(JRC \log_{10}(JCS)+\phi_0)}{r} \right) \]

\[ r = \frac{\tan(JRC \log_{10}(JCS)+\phi_0)}{\phi} \]

In view of the deeply weathered nature of the shales the base friction angle for the joint walls, \( \phi_b \), was assumed to be equal to that for the gouge given above. Furthermore as a result of the very soft rock consistency, the joint walls were estimated to have an average compressive strength, JCS, of 1 000 kPa. No variance was considered for this parameter. The average and standard deviation for the joint roughness constant, JRC, determined from several hundred field observations, amounted to 11.72 and 1.42 respectively. The average stress on the failure surface, \( \sigma \), was taken to be equal to 200 kPa.

The joint strength parameters were assumed to be represented by the tangent to the above function for \( r \) at the point \((\sigma, r)\). It can be shown that the intercept, \( c \), on the \( r \) axis and the inclination, \( \alpha \), of the tangent, in \( \alpha, r \) plot, are given by the expressions:

\[ c = \frac{\pi - \alpha \cdot JRC \cdot (1 + \frac{\sigma}{\phi_b}) \log_{10}e}{180} \]

\[ \alpha = \arctan \left( \frac{\sigma}{\phi_b} \right) \]

The distributions of these parameters were generated by randomly selecting values for JRC and \( \phi_0 \) from the given distributions and by substituting these in the above expressions. In the process 300 such selections were made. The resulting average and standard deviation for cohesion amounted to 20 kPa and 3 kPa, respectively. The corresponding values for friction angle amounted to 21° and 3.5°, respectively. A value of 0.1 was taken for the pore pressure parameter \( r_p \).

Three hundred values were selected randomly from each of the cohesion and friction angle distributions defined. For each selection of values a factor of safety was determined for the geometry of the failure surface according to the Janbu19 simplified method. The probability of failure was determined as the ratio of the number of factors of safety less than unity to the total number of factors calculated. The resulting probability of failure was calculated to be 75 per cent, which fits the significance scale proposed in Table 1.

Open cast mine

A plan and section of the rock slope in this example is shown in Fig 6. The slope is composed of a vertically-sided dolerite capping some 25 m deep, overlying a shale horizon 75 m deep and a lava horizon of considerable depth. The lava horizon is vertically inclined and is intersected at a depth of approximately 60 m below the shale contact by a 50° adversely inclined fault plane. The vertical front of the lava is

![Fig 5: Typical cross-section of unstable slope in rock cutting on construction works](image)

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butressed by waste rock. The dolerite and shale horizons are vertically jointed and are not the primary sources of instability. The fault plane, which is filled with a 1 m thick gouge, is the main cause of movements of the overlying material.

The characteristics of several thousand joints in the area which affect the slope were recorded from surface and underground surveys. The location and properties of the major fault plane through the lava horizon were determined from extensive underground observations and from small-scale laboratory tests respectively.

The fault plane can be considered to extend into the overlying shales in a step-jointed manner. The waste rock debris in front of the unstable block contributes to its stability and controls its rate of movement. The probability of failure of the block was determined for various heights of the waste rock in front of it and for various heights of the ground water table. In all cases the dolerite capping was assumed to impose a uniformly distributed vertical load on the shale horizon. It was furthermore assumed that the block was bounded laterally by two parallel vertical planes on which shearing resistance was mobilised during the slow forward progression of the block.

The restraining action on these planes was simulated by increasing the cohesion and friction angle of the underlying fault plane to the values of the competent underlying shale. The parameters $A$, $\alpha$, $\phi$, and $c$, were respectively assumed to denote the area of the fault plane in the lava, the average normal stress to this area, and the friction angle and cohesion of the area ($\alpha$, $c$,). The symbols $A$, $\alpha$, $\phi$, and $c$, were respectively assumed to denote the corresponding parameters of the step-jointed continuation of the fault plane in the shales, $i=2$, the lateral bounding planes in the lava, $i=3$, and the lateral bounding planes in the shales, $i=4$. Subject to these definitions the resisting force, $RF$, mobilised on the potential failure plane demarcating the bottom of the block, can be shown to be given by expression (i) in Appendix A.

To determine the significance of the parameters $A$, $\alpha$, $\phi$, and $c$, were determined from a considerable number of small scale laboratory and in situ tests. The distributions for $c$, and $\alpha$, were determined from a considerable number of small scale laboratory and in situ tests. The parameters $c$, $\alpha$, $\phi$, and $c$, were respectively assumed to denote the area of the fault plane in the lava, the average normal stress to this area, and the friction angle and cohesion of the area ($\alpha$, $c$,). The symbols $A$, $\alpha$, $\phi$, and $c$, were respectively assumed to denote the corresponding parameters of the step-jointed continuation of the fault plane in the shales, $i=2$, the lateral bounding planes in the lava, $i=3$, and the lateral bounding planes in the shales, $i=4$. Subject to these definitions the resisting force, $RF$, mobilised on the potential failure plane demarcating the bottom of the block, can be shown to be given by expression (i) in Appendix A.

The distribution of the normal stress parameter $A$ was considered as a coefficient which, when multiplied into the stress parameters for the fault plane, would represent the restraining effect of the lateral bounding planes in a plane failure analysis. The stresses $\sigma_A$, $\sigma_B$, $\sigma_C$, and $\sigma_D$, were determined from an appropriate axisymmetric stress analysis. The adjusted stress parameters for the fault plane in the lava, subscripted by $m$, and the step-jointed extension of the fault plane in the shales, subscripted by $s$, can be shown to be given by expressions (ii) in Appendix A.

The fault plane in the lava horizon comprised a deeply weathered material varying from 0.5 to 1.0 m in thickness. Distributions for $c$, and $\alpha$, were determined from a considerable number of small scale laboratory and in situ tests. The distributions for $c$, and $\alpha$, were determined from a considerable number of small scale laboratory and in situ tests. The parameters $c$, $\alpha$, $\phi$, and $c$, were respectively assumed to denote the area of the fault plane in the lava, the average normal stress to this area, and the friction angle and cohesion of the area ($\alpha$, $c$,). The symbols $A$, $\alpha$, $\phi$, and $c$, were respectively assumed to denote the corresponding parameters of the step-jointed continuation of the fault plane in the shales, $i=2$, the lateral bounding planes in the lava, $i=3$, and the lateral bounding planes in the shales, $i=4$. Subject to these definitions the resisting force, $RF$, mobilised on the potential failure plane demarcating the bottom of the block, can be shown to be given by expression (i) in Appendix A.

Ten potential loci for the extension of the failure surface into the adjoining shale and waste rock horizons were selected arbitrarily. Two hundred sets of values were selected randomly from the generated distributions for the strength parameters in respect of each of the ten potential failure surfaces. For each selection of values a factor of safety was determined for the unstable block according to the Janbu simplified method. The probability of failure was determined as the ratio of the number of factors of safety less than or equal to unity, to the total number of cases considered.

A plot of the probability of failure against the width of the block for the various cases considered is shown in Fig. 7. According to the principle of minimum potential energy, the actual width of the block was taken as that which corresponded to the maximum probability of failure. This dimension was in all cases approximately 80 m. A summary of the results is given in Table 4. The effective unstable zone, shown in Fig. 5, along which relative shearing took place, is approximately 70 m wide on average. This dimension corresponds remarkably well with the theoretically determined width of the unstable zone of approximately 80 m.

The relatively high probabilities of failure for all cases corresponding to a high water table confirmed a very early decision to dewater the affected slope of the mine. The results further illustrate the relative sensitivity of the stability of the slope to changes in the level of the waste rock against which the unstable block abuts. The main contribution by the waste rock to the stability of the block is the lengthening of the potential failure surface through the waste rock.

The results in this example confirm in principle the significance of the probability of failure proposed in Table 1.

Table 4: Maximum probability of failure for various depths of waste rock buttress and ground water table

<table>
<thead>
<tr>
<th>Depth of waste rock buttress</th>
<th>Depth of water table</th>
<th>Completely drained down</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Max phreatic surface through shales</td>
<td>Water table along shale to lava contact</td>
</tr>
<tr>
<td>Full depth of wedge above fault plane</td>
<td>$A$</td>
<td>$B$</td>
</tr>
<tr>
<td>Sixty per cent of above defined depth</td>
<td>$D$</td>
<td>$E$</td>
</tr>
<tr>
<td>Thirty per cent of above defined depth</td>
<td>$G$</td>
<td>$H$</td>
</tr>
<tr>
<td>Zero height of waste rock buttress</td>
<td>$J$</td>
<td>$K$</td>
</tr>
</tbody>
</table>

Symbols comprise legend to curves in Fig 7.

Conclusions

A scale is presented in which the significance of the probability of failure of slopes is given for a range of intervals between 0 and 100 per cent. It is based on the experiences and case studies of slope failures of a number of authors. The proposed significance scale is presented to assist in the development of a systematic probability-based design procedure for the experienced practising engineer.

The suitability of the scale is illustrated in terms of examples of soil, mine tailings and rock slopes, which essentially cover the full range of the scale from 0 to 100 per cent. The proposed scale will undoubtedly be refined as further applications are made.

It is illustrated how estimates of the probability of failure in respect of actual slopes can be used to estimate the actual cohesion and friction angle probability density functions by back calculation.

The major problem in slope engineering is to determine the strength parameters. The limitations in this regard are the usual scale effects in test samples, sample disturbance, inadequate equipment and methods of testing, and limited numbers of tests. Despite these shortcomings it is illustrated that the strength parameters obtained for soil on rock slopes from small scale tests or from field surveys, as the case may be, are sufficiently reliable to calculate the probability of failure in terms of the
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THE CIVIL ENGINEER in South Africa — January 1983
proposed scale.

It should be emphasized that the class intervals in the proposed scale correspond to the level of sophistication to which the input data for the relevant calculations are usually collected. The probability of failure is related to the scatter in the input parameters. If any attempt is made to reduce the usual scatter inherent in the data, by specially skilled engineering judgment¹° or by unusually sophisticated means of testing, the class interval boundaries in the proposed scale should be adapted accordingly. This applies especially to the lower categories of probability of failure.

Acknowledgements

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References


Appendix A

Expression (I):

\[ RF = \frac{A(c_i + o_i \tan \theta_2) + A(c_j + o_j \tan \theta_1)}{A(c_i + o_i \tan \theta_2) + A(c_j + o_j \tan \theta_1)} \]

Expression (II):

\[ c_m = k_c, \]

\[ e_m = \arctan(k_c), \]

\[ o_m = \arctan(k_c), \]

where

\[ k = 1 + \frac{A(c_i + o_i \tan \theta_2) + A(c_j + o_j \tan \theta_1)}{A(c_i + o_i \tan \theta_2) + A(c_j + o_j \tan \theta_1)} \]

Expression (III):

\[ c_i = \frac{k}{k+1}, \]

\[ e_i = \arctan(k), \]

\[ o_i = \arctan(k), \]

where

\[ i = 2, 3, 4 \text{ respectively correspond to the various planes forming the sides of the block as defined.} \]

Discussion

Written discussion on the preceding paper will be accepted until 31 March 1983. This, together with the author's reply, will be published in the October 1983 issue of The Civil Engineer in South Africa, or later. Such a discussion which must be submitted in duplicate, should be in the first person present tense and should be typed in double spacing. It should be as short as possible and should not normally exceed 600 words in length. It should also conform to the requirements laid down in the Notes for the Guidance of Authors and Contributors as published in the September 1974 issue of The Civil Engineer in South Africa.

Overseas contributors

For the convenience of overseas contributors only, the closing date for discussion will be extended to 15 April 1983 upon a receipt of a request together with an assurance that the material will be received by the Institution by that date. No request for any further extension can be considered.

Reference

Whenever reference is made to the above papers this publication should be referred to as The Civil Engineer in South Africa and the volume and date given thus: C\v Eng S Afr, Vol 25, No.1, 1983.
The history of Pfa in South Africa

The engineers who have been concerned with the promotion of Pfa in South Africa are mainly concrete technologists and understand well the problems encountered in making consistent concrete and concrete products. Little was known about the locally produced Pfa. This is not surprising, as it was only in the last decade that the modern pulverised-coal burning power stations, came into existence in South Africa.

Local engineers therefore had to become thoroughly acquainted with Pfa, before they could promote it. Thousands of tests were made and scientific programmes of sampling were carried out at power stations suitably equipped in terms of operational design, particularly in respect of the mills, furnaces and electrostatic precipitators.

Samples were sent overseas for appraisal by Scientific institutions and Pfa marketing concerns. It was found that the South African produced Pfa was of an exceptionally high quality in comparison with Pfa produced and marketed in other countries.

Experience gained in the early years enabled the engineers to correlate those factors contributing to quality Pfa, enabling them to develop techniques to control the selection of the material required and to introduce equipment for its extraction.

Reason for good quality

There are several reasons why the quality of South African Pfa is so good. The new power stations are designed to achieve the highest practical utilisation of the fuel. Each step of the process is continuously monitored, with key operations being automatically adjusted by computers within extremely fine limits. The ash produced, which is separated from the gas stream by the electrostatic precipitators, reflects the efficiency of the system.

The new power stations are supplied with coal from nearby sources. Unless something untoward occurs, this arrangement will continue for the operational life of the station.

Pfa is basically the unburnable mineral constituents of the coal — the oxides of silicon, aluminium, iron, magnesium, calcium and some sodium, potassium and titanium. The particular conditions under which combustion takes place in these modern boilers brings about some unique changes in the composition of these constituents, imparting special characteristics to the Pfa, such as its pozzolanic activity and the spherical configuration associated with its typical mobility.

Coal supplied to these power stations is a low-grade bituminous type with insignificant sulphur content and little or no quicklime. Coal deposits of this type are plentiful and therefore the ash produced can be expected to have the same constant composition.

CONSISTENCY

Pfa is one of the most consistent ingredients in concrete.

What is Consistency?
Consistency of a material is normally defined as meeting certain properties (physical and chemical) within predetermined limits. Whilst this facilitates some control, it does not necessarily guarantee consistent performance in the concrete. Yet this is the only consistency the end user is interested in.

Selection is the Key
Consistency is achieved by selecting material from one source only to much more exacting limits than the standards require.
Selected Pfa easily meets established standards as well as satisfying the end user's requirement of consistent performance.

Selection procedure
The steps taken in the selection process are shown in the table overleaf.

Contributing factors

There were a number of factors contributing towards the successful selection of quality material. One of the most important was to establish really effective liaison between power station personnel and the staff supervising the extraction plants. As long as the station, more...
precisely the particular boiler, from which the ash is being extracted, is functioning normally, the ash will be of consistent quality. Liaison is needed to warn the extraction plant personnel if and when anything is wrong, even temporarily, with the process.

Although such liaison is a key factor in the operation, it is not foolproof and may not be timeous enough to prevent some degree of contamination. Consequently, constant monitoring of the material, at frequent and regular intervals is now standard procedure. Samples are tested in the site laboratories established whenever extraction plants have been installed. A central laboratory regularly checks these samples and carries out the more sophisticated routine analyses and development work.

Material to waste dams

An aspect of inestimable value to the operation of extracting and marketing Pfa, which is quite unique to the operation, is the luxury of being able to divert any sub-quality material to the waste dams. The flexibility of the plant, together with the vast quantity of material from which to select, ensures that continuity of quality can be achieved merely by switching to another boiler. A modern power station produces some 3 million tons of Pfa per annum. A high percentage of this material will conform with our specifications and will therefore provide a continuous source of material for many customers who can be supplied by rail or road on an economic basis. The fact that a particular customer can be supplied with material from a single source is an aspect of the greatest possible significance. It is an absolute guarantee of consistency which no other ingredient in concrete can offer for an unlimited period.

The original commitment by those concerned with the development of the use of pulverised fuel ash in South Africa is no longer an unrealised resolution. It was translated into the very real benefits which are being derived by technicians throughout the country, not only as the result of the inherent qualities of the material but equally due to its exceptional consistency.

Right: Pneumatic pumping unit connected to one of the seventy-two precipitator hoppers from which the best Pfa is selected.

Selection – Quality Control Chain

1. Select the best source, i.e. select the right power station.
   a. By chemical and physical analysis of the Pfa in accordance with ASTM specifications.
   b. More important by a large number of performance tests of concrete specimens containing Pfa from this source.

The source selection determines the principle performance characteristics of the Pfa (its reactivity and water reducing capabilities). More specifically, the source selection dictates:
   - The chemical composition within narrow bands for years in advance (coal source does not change).
   - The “production” process within extremely tight operating limits due to Escom’s highly sophisticated controls.

This “locking in” of the fundamental characteristics of Pfa is the first requirement of consistent performance in concrete. Hence each customer must be supplied from one source only.

2. Select optimum control level on fineness. (% retained on 45 micron sieve.)

This is a self-imposed control limit far exceeding the requirements of ASTM C618 and other international standards. Consistent fineness is the second key requirement to consistent performance in concrete.

3. Maintain fineness control below chosen maxima within a narrow band.

4. Maintain loss on ignition (L.O.I.) (carbon content) below chosen maxima.

5. Control performance in concrete.

Periodic performance tests in concrete specimens ensure that the basic characteristic of the Pfa has not changed.

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