Computer analysis of the time-dependent behaviour of concrete structures

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Synopsis

Due to the nature or construction sequence of certain concrete structures it is necessary to include time-dependent effects in the analysis. This involves a considerable amount of numerical work which has to be done by computer. A method which can be used effectively with the stiffness method of structural analysis was coded and used for the analysis of segmentally erected bridges. Creep and shrinkage of concrete as well as material changes and temperature effects are taken into account. Efficient use of computer memory is obtained by storing the entire stress history in only a few variables at each integration point. A significant improvement in the results was obtained when the time-dependent effects were included in the analysis of a segmentally erected bridge. The method is suitable for a wide variety of structures.

SAMEVATTING

As gevolg van die aard van die konstruksievolgorde van sommige betonstrukture is dit noodsaaklik dat tyd-afhanklike faktore deur die analyse ingesluit word. Dit verg heelwat numerieuse werk wat met ’n rekenaar gedoen moet word. ’n Metode wat op effektiewe wyse saam met die styfheidsmethode van strukturaanaloës gebruik kan word is programmeer vir die analyse van brûe opgerig deur middel van segmentkonstruksie. Die kruipe en krimp van beton asook materiaal veranderings en temperatuurwisselings is in aanmerking geneem. Effektiewe gebruik van rekenaargereedse word verkry deur die spanningsrekord in net ’n paar veranderlikes by elke integrasiepunt te stoor. ’n Betekenisvolle verbetering in die resultate vir die analyse van ’n segmentbrug is verkry deur tyd-afhanklike faktore in te sluit. Die metode is geskik vir ’n wyse verskrekenheid strukture.

Introduction

In many concrete structures built today the time-dependent behaviour of the material can no longer be neglected. Two of the best examples are nuclear pressure vessels and segmentally erected concrete bridges where deflections and stresses which are functions of time have to be controlled. In this paper the influence of creep and shrinkage of concrete is discussed and it is then shown how a computer analysis can be done to take these effects as well as temperature changes and material changes into account. An example is given of a practical application of the method.

Creep of concrete

Definition

Creep is defined as the increment of deformation of a material under sustained load. Creep therefore does not include the instantaneous elastic deformation. The main factors affecting creep of concrete are compressive strength, age at loading, the type of aggregate, ambient relative humidity, temperature, size of the specimen, stress and duration of the applied stress. The analytical method should take these effects into account.

Available methods

Several analytical models have been proposed. Bazant\(^1\) compared a few approximate linear methods and showed that the Age-Adjusted Effective Modulus Method\(^2\) can be quite accurate. Danon and Gamble\(^3\) used the Rate of Creep Method which also gave satisfactory results. Both of the methods mentioned neglect the previous stress history and they are in general applicable only to statically determinate structures. Creep deformations of concrete can be predicted with reasonable accuracy if the concrete is considered to be a linear viscoelastic material. The response of the material step function inputs can be superimposed. The main problem then is to keep a record of the stress history in order to determine the current state of the concrete.

Several researchers have dealt with this problem but only an outline of the most suitable methods will be given. Reviews of the available analytical methods are given by Bazant\(^1\) and Kabir\(^4\). Kabir also showed how the ACI provisions\(^5\) can be used to find the necessary input material data for cases where experimental data is not available.

Zienkiewicz and Watson\(^6\) developed a method in which the necessary stress and strain history record is kept by retaining only the running sums of certain expressions. Small time steps are used during which the stresses are assumed to remain constant. At the end of the interval the incompatibility resulting from the creep strain change during the interval is corrected by an elastic solution. Loads are assumed to be applied at the end of the step when the instantaneous stress can be added to the stress change needed to restore compatibility. The elastic solution is most conveniently obtained by means of a stiffness method which permits any loading or initial strain system. Fig 1 illustrates the relationship between stress changes at the end of a time step and the corresponding creep strain change. Elastic strain is not included in Fig 1.

Bazant and Wu\(^7\) developed a method which is a further refinement of the same principle. The creep function is presented in the form of a Dirichlet series and a method was developed to identify it from known data. The creep function is given by

![Figure 1: Variation of stress and creep strain with time](image-url)

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simplifies the analysis greatly. \( D_p \) does not vary with time which means that the element stiffness matrix can be written as:

\[
K_n = E_n r_p B T D_o B D_p V
\]

(3.20)

\[
K_n = E_n S_0
\]

(3.21)

The matrix \( K_O \) is computed only once for an element and then multiplied by \( E_n \) to obtain the element stiffness matrix at time \( t_p B \) is a matrix relating the assumed displacement parameters to nodal displacements.

Note that when the structure is not erected in stages and the material properties are the same throughout, the same that was done here at element level can be done at structure level. It is in fact easier to solve the set of equilibrium equations

\[
K_o \Delta I_n o = \Delta R_n
\]

\[
\Delta I_n = \Delta I_n o e^n
\]

The triangularization of \( K_o \) needs to be done only once for the entire analysis and only the reduction of the load vector and back substitution to get \( \Delta I_n o \) have to be repeated for each new load vector \( \Delta R_n \).

It is obvious that for a structure with just one set of material properties and one configuration, each analysis step of a time-dependent analysis can be done with relatively little additional effort compared to that needed for analysing additional load cases in a time-independent analysis.

Poisson’s ratio varies from 0.15 to 0.25 and does not have a significant influence. Many uncertainties are still involved as far as the variation of \( v \) with time is considered. It is convenient to use a single value of \( v \) for each material throughout the time-dependent analysis. Normally a constant value of 0.18 is assumed. A value of 0.20 is recommended by the CEB.

The inelastic strain vector

The creep strain increment, shrinkage strain increment and the temperature strain increment are added to the virtual work expression and form part of the load vector that is assumed to be applied at time \( t_p \). These three strain increments are grouped together in the incremental inelastic strain vector:

\[
\Delta \varepsilon \varepsilon_n = \Delta \varepsilon_c + \Delta \varepsilon_s + \Delta \varepsilon_t
\]

where

\[
\Delta \varepsilon_c = \text{incremental creep strain vector}
\]

\[
\Delta \varepsilon_s = \text{incremental shrinkage strain vector}
\]

\[
\Delta \varepsilon_t = \text{incremental temperature strain vector}
\]

The increment in strain due to the change in elastic modulus \( \Delta \varepsilon_A \) does not contribute to the element load vector. It has however to be included in the constitutive relation. The vector \( \Delta \varepsilon_n \) is therefore added to \( \Delta \varepsilon_n \) after computation of the element load vector.

The creep strain increment and the hidden state variables

The creep strain increment for time step \( n \) is computed from

\[
\Delta \varepsilon_n = \frac{1}{n} \sum_{i=1}^{N} \varepsilon_i^c \varepsilon_i (t - e^{-\lambda_i}) (T_n - t) \Delta t_n
\]

where \( \varepsilon_i^c = 1 \), \( m \) are the hidden state variables.

The name ‘hidden state variables’ is credited to Bazant. Although the method Bazant used is somewhat different from that developed by Zienkiewicz and Watson, these variables fulfill the same function. The entire previous history is contained in these variables. The derivation of the expression for the hidden state variables \( \varepsilon_i^c \) is given in detail by both Zienkiewicz and Watson and Kabir.

Summary

To summarize the procedure presented here, the following steps show how it will be applied to one element of a structure.

1. Set up the necessary material properties:
\[ m \text{ = number of Kelvin models} \]
\[ t_0 \text{ = time of casting} \]
\[ f_{28} \text{ = 28 day compressive strength} \]
\[ S_i \text{ = slump of the concrete} \]
\[ S_j \text{ = minimum size of any side of the element} \]
\[ \varepsilon \text{ = ultimate shrinkage strain} \]
\[ \nu \text{ = Poisson’s ratio} \]
\[ \alpha \text{ = coefficient of thermal expansion} \]
\[ \lambda_i \text{ = } i = 1, m \text{ - retardation times} \]
\[ \alpha_i \text{ = aging constants. Set up a set of creep curves for the concrete of different ages from which } \alpha_i \text{ can be determined.} \]

2. Initialize the following variables at each integration point
\[ \varepsilon_i^* = i = 1, m \text{ - hidden state variables} \]
\[ Q \text{ = stresses} \]
\[ \xi \text{ = strains} \]
\[ \xi' \text{ = pseudo inelastic strains} \]

3. Compute the elastic modulus \( E_1 \) for the time \( t_1 \) when the element is attached to the structure and loaded for the first time.

4. Solve the equilibrium equations and compute the instantaneous stresses and strains due to this first loading. The procedure is the same as step 6 through 21 but \( \varepsilon^{*} = Q \) at this stage.

5. Compute \( H_n \) at the end of the next time step \( t_n \). Compute
\[ \Delta t_n = t_n - t_{n-1}. \]

6. Set up the temperature shift function \( \Phi \) and the aging parameters \( a_j(t_n) \).

7. Compute the creep strain increment
\[ \Delta \varepsilon = \sum_{i=1}^{m} \varepsilon_i^*(1 - e^{-\lambda_i \Phi/(T_{n-1} - \Delta t_n)}) \]

8. Compute the shrinkage strain increment \( \Delta \xi \).

9. Compute the temperature strain increment \( \Delta \xi \).

10. Compute the strain increment \( \Delta \xi = \xi_{n-1}/E_{n-1} - 1/E_n \) due to change in elastic modulus.

11. Compute the initial strain that will contribute to the load vector:
\[ \Delta \xi_n = \Delta \xi_{n-1} + \Delta \xi + \Delta \xi \]

12. Compute the contribution from the initial strains to the load vector:
\[ \Delta R_n = B^T D_n \Delta \xi_n \]

13. Compute the total pseudo-inelastic strain:
\[ \Delta \xi_n = \Delta \xi_{n-1} + \Delta \xi + \Delta \xi + \Delta \xi \]

14. Repeat steps 7 through 13 for all integration points.

15. Add all new externally applied loads to the load vector to get the total incremental load vector \( \Delta \eta \).

16. Set up the structure stiffness matrix \( K_n \) from the element stiffness matrices.

17. Solve for the incremental displacement vector \( \Delta \xi_n \) from
\[ K_n \Delta \xi_n = \Delta \eta_n \]

18. Compute the element displacement vector \( \Delta \xi_n \) from \( \Delta \xi_n \) if static condensation has been used.

19. For each integration point compute:

(a) The strain increment in time step \( n \)
\[ \Delta \xi_n = B \Delta g \]

(b) The total strain
\[ \xi_n = \xi_{n-1} + \Delta \xi_n \]

(c) The total stress
\[ \sigma_n = D_n (\xi_n - \xi_{n-1}) \]

(d) The stress increment in time step \( n \)
\[ \Delta \sigma_n = \sigma_n - \sigma_{n-1} \]

(e) If required the increment in internal displacements in time
\[ \Delta \xi_n = A q_n \]

where \( A \) is a matrix containing the assumed displacement functions.

(f) Update the hidden state variables
\[ \xi_i^{*}(n+1) = \xi_i^{*}(n) - \xi_i^{*}(T_{n-1}) \Delta \xi_n + \Delta \sigma_n \]

20. Select time \( t_n+1 \) and repeat steps 5 through 19 for all time steps.

**Applications**

The method described here was used in a computer program developed by me for the analysis of segmentally erected prestressed concrete box girder bridges\(^1\). It was found that the computing time needed to do the time-dependent part of the analysis was only one third of the total time needed to do a complete analysis including data generation, prestressing analysis and changing structural configurations.

In Fig 2 the elevation, cross section and erection sequence of the Corpus Christi Intercoastal Canal Bridge in Texas is shown. This bridge consists of a main span of 80 m with two side spans of 30 m each and was erected by means of the segmental erection procedure. Prior to its construction a comprehensive study was done at the Center for Highway Research at the University of Texas at Austin. The study included the development of an analysis computer program, SIMPLA2, and the construction as well as testing of a model of the Intercoastal Canal Bridge. In Fig 3 the prestressing tendons are shown.

Table 1 lists the various construction activities in more detail.

<table>
<thead>
<tr>
<th>Stage</th>
<th>Action</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-9</td>
<td>Addition of segments SS1 through SS9 and MS1 through MS9; stress tendons B1 through B9</td>
</tr>
<tr>
<td>10</td>
<td>Addition of segment MS10; cast closure strip</td>
</tr>
<tr>
<td>11</td>
<td>Stress tendon B10</td>
</tr>
<tr>
<td>12</td>
<td>Addition of segment SS10, stress tendon C4</td>
</tr>
<tr>
<td>13-15</td>
<td>Stress remaining C-series tendons in the order C3, C2, C1</td>
</tr>
<tr>
<td>16-19</td>
<td>Stress A-series tendons in the order A1, A2, A3, A4</td>
</tr>
<tr>
<td>20</td>
<td>Remove rotational support at main pier</td>
</tr>
<tr>
<td>21</td>
<td>Raise end support 33 mm to increase end reaction</td>
</tr>
<tr>
<td>22,23</td>
<td>Stress tendons A5, A6</td>
</tr>
</tbody>
</table>

Note: Refer to Figs 2 and 3 for segment and tendon numbers. Stage numbers refer to the actual construction sequence.

Brown, Burns and Breen\(^2\) used the program SIMPLA2 developed by them to do an analysis of the bridge at the construction stages of Table 1. Their analysis does not account for time-dependent effects. An analysis in which the time-dependent effects are taken into account was done by me using the method described here.

A comparison was made between the results obtained and those obtained by Kashima and Breen\(^3\) and Brown, Burns and Breen\(^4\) and agreement was found to be good with a significant improvement when time-dependent effects were considered. In Fig 4 the strain at the top of segment MS1 during erection is plotted for the three methods mentioned. In Fig 5 the deformed shape at different stages in the construction as predicted by means of the method described here is plotted. Results such as these can be used to plan the alignment of segments in order to obtain closure or to end up with a predetermined deformed shape.

**Conclusion**

The method described here was found to be suitable for computer analysis of time-dependent effects in concrete. Although it was used in a program for the analysis of segmentally erected bridges it can be used in conjunction with any other analysis of structures such as frames, plates, shells or solid finite elements. It was found that although the development of the program to include time-dependent effects was costly and more complicated.
than usual, the additional analysis cost of specific examples was only one third of the total analysis cost.

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References

Fig 2: Details of Corpus Christi Intercoastal Canal Bridge

Fig 3: Layout of prestressing tendons of the Intercoastal Canal Bridge

Fig 4: Total strain at top of segment MSI during erection

Fig 5: Deflection of Intercoastal Canal Bridge during construction