The minimum quantity of steel to limit the size of cracks in thin concrete walls

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Summary

Expressions are derived for the stresses developed in steel and concrete when a thin concrete wall suffers a fall in temperature. It is shown that the concrete is unlikely to crack unless there is some external restraint. When restraint is complete and a crack occurs there is a considerable reduction in the tension in the wall. Since it ignores this reduction, the simple formula in Appendix B of BS 5337 for the minimum proportion of steel is invalid.

BS 5337 : 1976

Reinforcement is usually placed in otherwise unreinforced concrete structures such as walls in order to limit the size of thermal or shrinkage cracks. For liquid-retaining structures it is particularly important that the size of cracks be as small as possible, as emphasized in the British Standard Code of Practice, BS 5337 : 1976.

In common with most other codes of practice for reinforced concrete, BS 5337 allows the minimum amount of steel for crack-control to be calculated as a proportion of the overall cross-sectional area of the concrete. The minimum proportion of steel in each face and in each of two directions at right angles must be 0.15 per cent for deformed high yield reinforcement or 0.25 per cent for plain mild steel reinforcement, i.e. a total of 0.3 per cent or 0.5 per cent respectively in each direction. This proportion is a little greater than the minimum quantity required by CP 110, for example, because of the importance of keeping down the size of cracks in liquid-retaining structures.

As an alternative to this simple method, BS 5337 offers a more detailed calculation in clause 15 and Appendix B, by which the size and spacing of cracks may be related to the type and proportion of steel, and to the expected changes in temperature and humidity. According to Appendix B of BS 5337, the minimum proportion of steel is first calculated as the ratio of the tensile strength of immature concrete to the strength of the steel. Values suggested in BS 5337 for the tensile strength of concrete (at the age of three days) are 1,15 MPa for 25 grade concrete and 1,3 MPa for 30 grade concrete. The strength of steel is to be taken from Table 3 of CP 110, i.e. 250 MPa for mild steel and 410 MPa for hot-rolled high-yield steel, with the exception that for other steels the value used must not be greater than 425 MPa. The assumption made when the minimum proportion of steel is calculated by the method given in Appendix B of BS 5337 is that after the crack has occurred the steel alone bears the force carried by the concrete before the crack occurred. It is therefore assumed that if the proportion of steel is less than the ratio of tensile strengths of concrete and steel then the steel will yield and cannot adequately limit the size of the crack. It is the purpose of this study to examine the validity of that assumption.

Early thermal stress

The earliest risk of cracking occurs when the concrete cools after the rise in temperature caused by hydration of the cement. Temperatures observed by C D Turton in walls up to 0.5 m thick during the first few days after casting are shown in Fig 1. In walls of this thickness the variation of temperature, and thus of stress, through the wall may generally be expected to be small enough for temperature and stress to be considered as uniform for practical purposes. Such walls may therefore be considered as thin walls according to the terminology of BS 5337. It should be noted that if there were a steep temperature gradient in thin structures, the assumption made in Appendix B of BS 5337 would not hold. Fig 1 shows that the peak temperature occurred within the first 24 hours and that a fall in temperature of as much as 30°C occurred within the first three days. J G Hunt has shown experimentally that fully restrained concrete cannot sustain a fall in temperature of more than about 4 to 15°C, depending on the type of aggregate, through its influence on the coefficient of thermal expansion of the concrete. Fully restrained concrete subjected to the changes in temperature illustrated in Fig 1 is therefore likely to crack within three days irrespective of the type of aggregate used.

Degrees of restraint

A high degree of restraint of concrete during its early life is only likely to occur when, for example, concrete is cast on to a lower portion of a wall which is already cool. The effective restraint may be expected to be less, the higher construction proceeds above the base of the wall. For concrete remote from the contact surface with the previous lift, the only restraint may be that provided by the reinforcement itself. Since the temperature of the reinforcement may be assumed to be close to that of the concrete at any time, tensile stress could only arise if the coefficient of thermal expansion of the concrete is greater than that of the steel.

If the steel provides the only restraint, the strains developed in the concrete and steel may be estimated as follows. If the concrete were free to contract when the temperature fell by an amount T, the contraction would be \( a_c \ T \) per unit length, where \( a_c \) is the coefficient of thermal expansion of the concrete. Similarly the steel would contract by an amount \( a_s \ T \). Since the concrete and steel are bonded together, a tensile stress will be induced in the concrete and a compressive stress in the steel so that both contract an equal amount. If \( e_c \) and \( e_s \) are the strains developed in the concrete and steel respectively as a result of these forces

\[
(a_c - a_s) \ T = e_c + e_s
\]

If \( f_c \) is the tensile stress induced in the concrete, then

\[
f_c = e_c E_c
\]

where \( E_c \) is the secant modulus of elasticity.

The tensile force in the concrete is equal to the compressive force in the steel, hence

\[
f_c A_c = f_s A_s
\]

where \( A_c \) and \( A_s \) are the cross-sectional areas of the concrete and steel respectively. Putting \( A_s/A_c = p \),

\[
f_c = pf_s
\]

Hence Eqn 1 may be written

\[
(a_c - a_s) \ T = pf_s E_c + f_s E_s
\]

In Hunt’s investigation concrete made with flint gravel or
quartzite aggregate, which had the highest coefficient of thermal expansion of $14 \times 10^{-6}/\text{°C}$, sustained the smallest fall in a tensile temperature of about 5° C, when the strain reached $70 \times 10^{-6}$. Taking the tensile strength of the concrete at three days to be about 1,2 MPa, as suggested in Appendix B of BS 5337, gives a secant modulus of elasticity at three days of about 17 GPa.

Substituting the values mentioned, 10 x $10^{-6}/\text{°C}$ for $a_s$ and 200 GPa for $E_S$, in Eqn 2 gives

$$47 \times 10^{-6} = f_s \left( \frac{\rho}{17000} + \frac{1}{200000} \right)$$

$$47 = f_s / (59 \rho + 5)$$

For a typical value of $\rho$ of 0,5 per cent (the simple method of BS 5337 requires 0,25 per cent at each face) it is apparent that the value of $\rho$ has little influence on the relationship between $f_s$ and $E_S$. For the maximum fall in temperature recorded by Turton of 30° C, the compressive stress induced in the steel would be about 23 MPa and the strain in the steel 23/200 000 = 115 x $10^{-6}$.

From Eqn 1 the sum of the strains in concrete and steel as a result of a fall in temperature of 30° C is 120 x $10^{-6}$. The strain in the concrete would therefore be $5 \times 10^{-6}$ which is equivalent to a tensile stress of 17 000 x $5 \times 10^{-6}$ or less than 0,1 MPa, which is less than one tenth of the estimated tensile strength. No danger of cracking could therefore be expected during the early life of the concrete as a result of temperature stresses when the sole restraint is provided by the reinforcement.

At a later stage shrinkage may lead to tensile stress in concrete restrained only by reinforcement. If the free shrinkage strain is denoted by $e_{cs}$ the equivalent form of Eqn 2 is

$$e_{cs} = \rho f_{se} E_S + f_s / E_S$$

For a shrinkage strain of 300 x $10^{-6}$ a modulus of elasticity of concrete, of say, 30 GPa and proportion of steel 0,5 per cent, this equation gives a stress in the steel of less than 60 MPa and a corresponding stress in the concrete of about 0,3 MPa. Thus even if the temperature stresses are combined with considerable shrinkage stress, it appears that the concrete could never crack solely through the restraint of steel in the normal proportions used for limiting the size of cracks.

**Full restraint**

Thermal or shrinkage cracks can therefore only be expected if there is some external restraint. The stresses in a fully restrained element may be estimated by reference to Fig 2. If $n$ cracks occur in an element of length $L$, then $L$ represents the average spacing of the cracks.

In estimating the distribution of stress along the length of the member after a crack has occurred, it is sufficiently accurate to assume a linear increase in stress in the concrete for a short distance on either side of the crack until a constant stress is reached. The width of cracks close to the reinforcement has been shown to be about 0,025 mm and has been taken as zero for the purposes of this discussion.

Fig 1 shows that high temperatures may build up before the concrete may be expected to be hard enough to offer any resistance to the expansion of the steel. It is therefore reasonable to assume that before cooling begins the stresses in the system are all zero. If, after cooling, $f_{s1}$ is the stress in the steel under uniform conditions away from a crack and $f_{s2}$ is the stress in the steel at a crack, the mean stress in the steel over the lengths of stress development is ($f_{s1} + f_{s2})/2$. If $\Delta l$ is the stress development length, the elongation produced by the stress in the steel over this length would therefore be

$$\frac{f_{s1} + f_{s2}}{2E_S} \Delta l = (f_{s1} + f_{s2})n \Delta l / E_S$$

The elongation in the portions of steel under constant stress would be

$$f_{s1} (mL - 2n \Delta l) / E_S$$

The contraction which would have occurred if the unit had been allowed to cool un restrained is $\alpha/n/L$, where $\alpha$ is the coefficient of thermal expansion of steel and $L$ is the fall in temperature. Hence

$$f_{s1} + f_{s2}) n \Delta l = f_{s1} (mL - 2n \Delta l) = E_S \alpha T L$$

This equation takes no account of any effect on the stress in the steel caused by a difference in coefficient of thermal expansion between the concrete and the steel. Eqn 3 is therefore only valid when these coefficients are the same. A more detailed analysis is, however, not necessary in order to establish principles.

The strains in the steel and concrete in the portion under uniform stress are the same. Hence, if $f_{c1}$ is the stress in the concrete in the portion under uniform conditions,

$$f_{c1}/E_C = f_{s1}/E_S$$

where $E_C$ is the modulus of elasticity of concrete, taken to be constant during the period of cooling up to the time the crack occurs.

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**Fig 1:** Temperature of concrete walls soon after casting

<table>
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<tr>
<th>Thickness</th>
<th>0.4 m</th>
<th>CEMENT CONTENT 450 kg/m³</th>
</tr>
</thead>
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<tr>
<td>Temperature (°C)</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Time After Placing (h)</td>
<td>10</td>
<td>20</td>
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<table>
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<tr>
<th>Thickness</th>
<th>0.45 m</th>
<th>CEMENT CONTENT 370 kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Time After Placing (h)</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>

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<table>
<thead>
<tr>
<th>Thickness</th>
<th>0.5 m</th>
<th>CEMENT CONTENT 225 kg/m³</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature (°C)</td>
<td>30</td>
<td>20</td>
</tr>
<tr>
<td>Time After Placing (h)</td>
<td>10</td>
<td>20</td>
</tr>
</tbody>
</table>
The force in the unit is constant throughout its length. Equating the force at the crack with that in the portion under uniform conditions gives

$$f_{s2}A_S = f_{s1}A_S + f_{c1}A_C$$

Putting $$A_S/A_C = p$$,

$$p(f_{s2} - f_{s1}) = f_{c1}$$

(5)

Substituting for $$f_{c1}$$ and $$f_{s1}$$ from Eqs 4 and 5 in Eqn 3 gives

$$p(f_{s2} - E_S\alpha T) = E_C\alpha T - \frac{f_{s1}}{E_S}$$

(6)

**Numerical example**

The distance, $$l$$, over which stress builds up to a uniform value may depend on many factors, such as number, cross-sectional area and type of steel bars. Glanville found that the shrinkage stress in concrete having a section of 150 x 150 mm built up over a distance between about 50 and 200 mm. The smaller the diameter of the bar and the larger the proportion of steel, the shorter the development length. These values appeared to be independent of the age of the concrete. For a small percentage of steel used for crack limitation, say 0.5 per cent, Glanville’s tests indicate that a build up distance of about 120 mm would be appropriate for a concrete thickness of 150 mm.

Little information is available about the spacing of thermal cracks under given conditions. An indication of the order of magnitude of the spacing may be gained from the measurements recorded by Hughes who found that the average spacing of cracks was 2.5 m in a wall having a mean thickness of 600 mm.

In Hunt’s investigation, concrete made with granite aggregate had a coefficient of thermal expansion at the middle of the range, 10 x 10^-6 per °C. Failure occurred when the temperature fell by 8°C at a strain of 80 x 10^-6. For a tensile strength at three days of about 1.2 MPa the secant modulus of elasticity of the concrete up to failure would thus be about 15 GPa.

Substituting these values in Eqn 6 gives

$$p(f_{s0} - 1.2) = 1.2 - 0.0036 f_{s0}$$

(7)

The minimum proportion of steel, defined as that at which the steel would yield, is given by Eqn 7 if $$f_{s0}$$ is equal to the yield stress. Taking the yield stress of mild steel as 250 MPa gives a minimum proportion of steel of

$$\frac{1.2 - 0.9}{234} = 0.3/234 = 0.0013$$

or 0.13 per cent, which is much less than the minimum given by the formula in Appendix B of BS 5337 of about 1,2/250 = 0.0048 or 0.48 per cent.

For high yield steel, if $$f_{s0}$$ is equal to 410 MPa, the minimum proportion of steel given by Eqn 7 is negative, a result which implies that the yield stress in the steel would never be reached under the given conditions and that there is therefore no minimum proportion of high yield steel, as defined above. The minimum proportion of high yield steel given by the method of BS 5337 is about 1,2/410 = 0.003 or 0.3 per cent.

In practice there may be considerable divergence from some of the values used. For walls thicker than 150 mm the value of $$l$$ is likely to be greater than the value quoted. Also for concrete weaker than the mature concrete in Glanville’s tests, $$l$$ is likely to be greater than his experimental value. The spacing of cracks quoted in Hughes’ observations may be an overestimate since all cracks narrower than 0.04 mm were ignored in that examination. Both these factors would tend to give an even lower estimate of the minimum proportion of steel.

If $$T$$, the fall in temperature to cause cracking, were greater, or the tensile strength of the concrete at the time of fracture and hence the modulus of elasticity calculated from it were higher than the values quoted, the estimated minimum proportion of steel would be greater.

**Reduction in force when a crack occurs**

If the conditions assumed for the calculation are typical, then it is clear that the formula in Appendix B of BS 5337 overestimates the minimum proportion of steel, particularly for high yield steel. The reason it does so becomes apparent if an estimate is made of the forces before and after a crack occurs.

Taking the values in the example above, the force in the concrete just before the crack occurs is

$$A_C E_C \times \text{strain} = A_C \times 15,000 \times 80 \times 10^{-6} = 1.2A_C$$

The force in the steel just before the crack occurs is

$$A_S E_S \times \text{strain} = A_S \times 200,000 \times 80 \times 10^{-6} = 15pA_C$$

The combined force in the unit just before the crack occurs is thus

$$A_C (1.2 + 16p)$$

After the crack has occurred the force in the unit is equal to the force in the steel at the crack or $$A_S f_{s2} = pA_C f_{s2}$$.}

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**Fig 2: Stresses in cracked restrained concrete element**

**DIE SIVELE INGENIEUR in Suid-Afrika ~ September 1980**
Discussion on papers

Written discussion on this and following paper will be accepted until 15 November 1980. This, together with the authors’ reply, will be published in the May 1981 issue of The Civil Engineer in South Africa, or later.

Such written discussion, which must be submitted in duplicate, should be in the third person present tense, and should be typed in double spacing. It should be as short as possible and should not normally exceed 600 words in length. It should also conform to the requirements laid down in the ‘Notes for the Guidance of Authors and Contributors’ as published in the September 1974 issue of The Civil Engineer in South Africa.

Overseas contributors

For the convenience of overseas contributors only the closing date for discussion will be extended to 30 November 1980 upon a receipt of a request together with an assurance that the material will be received by the Institution by that date. No request for any further extension can be considered.

Reference

Whenever reference is made to the above paper this publication should be referred to as The Civil Engineer in South Africa and the volume and date given thus: Civ Engr S Afr, Vol 22, No. 9, 1980.

Papers requested for
SA symposia

SA Computer Symposium

A symposium on research in theory, software and hardware will be held at the CSIR Conference Centre, Pretoria, from 28 to 29 October 1981. This symposium is intended to be a broadly based presentation of research in the whole field of computer science and papers or communications are invited on the following topics: information processing, computer science theory, program languages, operating systems, computer architecture, database systems and performance evaluation. The deadline for submission of draft papers is 5 January 1981.

If you intend presenting a paper or a communication or attending the symposium, please advise The Symposium Secretariat S. 231, CSIR, P.O. Box 395, Pretoria, 0001, before 31 August 1980.

CONSAS 82

The next conference of Southern African Surveyors (CONSAS 82) takes place at the Witwatersrand University, Johannesburg from 30 January to 6 February 1982, followed by an Engineering Surveying Conference to be held under the auspices of CONSAS 82 and under the banner of FIG Commission 6 on 8 to 9 February.

Opening addresses will be delivered by eminent personalities in the mining world, the government and from large industrial organizations associated with the development of the Republic’s resources. In addition to prominent local speakers it is hoped to have a dozen or more overseas speakers of international repute to deliver papers.

Persons who may wish to deliver papers at any of the two conferences and covering any aspect of the survey discipline should communicate with the CONSAS 82 Conference Secretary, P.O. Box 4846, Johannesburg, 2000.