Slipping of PTFE bearings on long bridges

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Synopsis
A study is made of the slips which occur in PTFE (or Teflon) bearings on long bridges with slender piers as a result of temperature change, creep and shrinkage. The static and dynamic coefficients of friction on PTFE bearings differ by up to a factor of two and this leads to 'stick-slip' behaviour. A mathematical model of the behaviour of the bearing is proposed and a numerical scheme for carrying out the calculation is presented. Results which are in broad agreement with observations are obtained.

Introduction
In recent years an increasing number of long highway bridges which are continuous over a number of spans have been constructed. These bridges are usually of the form of elevated freeways or viaducts with slender piers. Such long bridges are subject to large temperature, shrinkage and creep strains which must be accommodated by the elastic deformation of the piers or the sliding of suitably designed bearings. Fig 1 shows a typical bridge configuration. The 'pinned' piers have no provision for sliding between the pier and the deck, whereas the 'free' piers incorporate bearings which permit sliding.

In modern practice extensive use has been made of bearings which permit sliding on an interface between stainless steel and a polytetrafluoroethylene (PTFE or Teflon) sheet. A typical bearing is shown in Fig 2. This particular bearing permits unidirectional horizontal sliding on the flat surfaces and rotational sliding on the spherical surfaces.

Inelastic longitudinal strains lead to a number of problems which require analysis and a detailed review of the field has been given by Reynolds and Emanuel[1]. A particular aspect of the behaviour of PTFE bearings which does not appear to have received attention, however, is the difference between the static and dynamic coefficients of friction. Typical values of the coefficient of friction are 0.03 to 0.06 for the dynamic value and 0.035 to 0.07 for the static value. A considerable degree of variation can be expected: the coefficient depends on the normal load and the static value may increase with time. Static coefficients may be twice as large as the dynamic value.

The difference between the static and dynamic coefficients leads to a 'stick-slip' phenomenon, in that as the temperature increases or decreases monotonically, sliding on the bearing occurs discontinuously with time. The discontinuous behaviour is made up of a number of cycles in which the pier or supporting column deforms elastically first and then slip occurs on the bearing suddenly.

Fig 3 shows the results of tests carried out on a PTFE bearing on the Cape Town Foreshore Freeway[2]. In this case the bearing was mounted on a steel frame which in turn rested on the pier; the steel frame acted as a force transducer. The displacement shown as a function of time in Fig 3 is the sum of the elastic displacements of the transducer and the horizontal slipping of the bearing. The discontinuous motion can be seen quite clearly.

In practice, longitudinal temperature, shrinkage and creep effects on bridge superstructures coupled with the 'stick-slip' nature of the bearing movement pose a twofold problem to the design engineer. Firstly, sufficient allowance has to be made for the expected movement at expansion joints in the bridge deck and anticipated movements of the deck on its 'free' bearings. It has been the practice to avoid detailed evaluations of these movements and to design for conservative values.

The second consideration involves the evaluation of maximum forces developed in the bridge structure (mainly in the columns and 'fixed' bearings) due to friction in the 'free' bearings. Since the bearings tend to slide at different points in time, the direction of all frictional forces can conceivably be in the same direction at the same time. This can have major implications in bearing and column design and can only be assessed to any degree of accuracy by a detailed analysis.

It is the purpose of this paper to construct a model which accommodates the different static and dynamic slip values, and to show that detailed numerical solutions can be obtained. The problem is relatively straightforward, as will be seen, but has several interesting features which distinguish it from classical plastic problems. We shall concern ourselves only with longitudinal sliding but it will be evident that the rotational sliding effects can be treated in a similar manner.

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Fig 2: Typical PTFE bearing

Fig 1: Diagrammatic sketch of continuous girder bridge

Scale 1:1000
Mathematical model

The actual bridge, shown in Fig 1, is modelled in the manner shown in Fig 4. The bridge deck is taken as a straight line element and, since we consider temperature changes independently of any other loading on the bridge, axial displacements alone are permitted. The piers and bearings provide constraints on axial displacements at the node points. It will be assumed that the bridge deck is uniform between nodes and the node displacements are taken as the primary parameters of the problem.

Fig 4: Mathematical model for axial deformation

Let $\{u\}$ be the column vector of unconstrained nodal (deck) displacements. A typical deck element, which we shall refer to as a type D element, connects two adjacent nodes. The type D element is assumed to behave elastically with a constitutive equation of the form

$$\delta = \frac{N}{S} + \delta_0$$

where $\delta$, $N$ and $S$ are respectively the extension, axial force and axial stiffness. The term $\delta_0$ represents imposed inelastic deformation given as a function of time; it is basically the instantaneous due to temperature fluctuations although it can also include creep and shrinkage effects.

We shall assume that each node is supported by an element which will be designated as either type A or type B. The type A element models a PTFE bearing on a rigid abutment; these bearings will occur at the ends of the bridge if they occur at all. It is assumed that the type A bearing is rigid-perfectly plastic, with a characteristic axial force-extension relation shown diagrammatically in Fig 5(a) and described mathematically by

$$\delta = \delta_D$$

$$\delta_D > 0 \text{ if } N = +N_D$$

$$\delta_D = 0 \text{ if } -N_D \leq N \leq N_D$$

$$\delta_D < 0 \text{ if } N = -N_D$$

Axial forces such that $N > N_D$ and $N < -N_D$ are not permitted. In the type A bearing the difference between the static and dynamic coefficients of friction is ignored. A model which included the difference would be slightly more accurate, but would have little effect on the overall behaviour at the cost of considerable complication in the analysis. Inelastic deformation or slip in type A bearings will take place continuously in the most part and this is provided for in the rigid-plastic model.

The type B element models a PTFE bearing on a flexible pier. If we follow the physical behaviour as the temperature changes in the deck we see that motion of the node will be accommodated by elastic deforma-

Fig 5: Models of abutment and pier bearing

The first is the time scale of temperature change: this is very slow compared to dynamic effects in the bridge. We may hence consider that the changes which result from the sudden change in the interactive force from $N_S$ to $N_D$ can be considered to occur at constant temperature. When the force changes suddenly, the structure is no longer in equilibrium and the structure may accelerate. We make the further assumption that the time scales of dynamic response of the pier in question and the deck, supported on the remaining piers and abutments, are distinct. This permits us to consider separately the response of the pier, with the node fixed, and the response of the deck.

In considering the pier, we make use of the one degree of freedom model shown in Fig 5(b). The extension of the element is $\delta$ and the slip $\delta_D$ is an internal variable. The mass of the pier is taken as $M$ and the stiffness as $S$, and the yield and flow forces are $N_D$ and $N_D$ respectively. The terms $M$ and $S$ are taken as the effective mass and stiffness for a free vibration of a cantilevered pier in its first or fundamental mode. Suppose that the bearing is slipping for the first time. The axial force at the instant that slip is initiated is

$$S\delta = N_D, \quad \delta_D = 0$$

(3)

Fig 3: Observed behaviour of PTFE bearings on the Cape Town Foreshore Freeway

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Thereupon the force in the bearing drops and the equation of motion is
\[ M (\ddot{\delta} - \ddot{\delta}_p) + S (\dot{\delta} - \dot{\delta}_p) = N_D \]  
(4)

If we assume that the node does not move, \( \delta \) is fixed and has its initial value. Hence Eqn 4 becomes
\[ M \ddot{\delta}_p + S \dot{\delta}_p = S \ddot{\delta} - N_D = S \ddot{\delta}(0) - N_D = N_S - N_D \]
(5)

The solution of this equation is
\[ \delta_p = \frac{N_S - N_D}{S} (1 - \cos \omega t) \]
(6)

where \( \omega = S/M \). This solution will hold until \( \omega t = \pi \), when the force in the bearing must change sign. At this instant the bearing locks, since \( N \) will not reach the value \(-N_S\) required to initiate slip in the opposite direction, and \( \delta_p \) remains fixed at the value given by Eqn 6 with \( \omega t = \pi \).
\[ \delta_p = \frac{2(N_S - N_D)}{S} \]
(7)

This slip is assumed to take place suddenly, ie, rapidly compared to any other time scales of importance. Once the slip has occurred the system is still not in equilibrium and an oscillation may occur. We are not concerned with the details of this further oscillation, but only in the final equilibrium state when the oscillation has damped out. We have thus to solve an elastic problem to complete the analysis of the effects of a slip in a type B element.

Formally, each time the force in a type B element attains the value of \( N_S \) or \(-N_S\), we introduce a sudden change, a discontinuity in the time scale of the temperature fluctuation, in the slip \( \delta_p \) given by
\[ [\delta_p] = \frac{2(N_S - N_D)}{S} \]
(8)

where \( Sgm \) \( N = +1 \) if \( N = +N_S \) at the instant slip is initiated and
\[ Sgm \] \( N = -1 \) if \( N = -N_S \) at the instant slip is initiated.

We then carry out an elastic analysis of the structure as a whole to determine the discontinuities in nodal displacements and axial forces which will result from this change.

In our overall analysis, therefore, slip in type B bearings will appear as a series of discontinuous jumps leading to discontinuities in the node displacements and bar forces. This accords with observations of the actual behaviour shown in Fig 3.

Numerical formulation

We may now assemble the equations which govern the problem. The \((n \times 1)\) column vector \([u]\) is made up of the unstrained nodal displacements. Let there be \(m\) elements in total, comprising type A, B and D elements and leading to \((m \times 1)\) column vectors \([\delta]\), \([\delta_0]\), \([\delta_p]\) and \([N]\) representing respectively the extensions, imposed inelastic extensions, slips and axial forces in the elements. It is understood that the components of \([\delta_0]\) for type A and B elements are always zero, while the components of \([\delta_p]\) for type D elements are always zero.

A deformation matrix \([B]\) relates the extensions to the nodal displacements
\[ [\delta] = [B][u] \]
(9)

Since no external forces are applied at the nodes, the equilibrium equations can be written as
\[ [B]^T[N] = 0 \]
(10)

It is convenient to define a global stiffness matrix \([S]\) which is diagonal and made up of the element stiffness \(S\). These stiffnesses are not defined for type A elements which are rigid; nevertheless it is useful to carry a notion of stiffness although it is not actually used in the computation. The constitutive equation is then written in the form
\[ [N] = [S] \left( [\delta] - [\delta_p] - [\delta_0] \right) \]
(11)

Combining Eqns 9, 10 and 11 we have
\[ [B]^T \left[ [S] + [B] \right] [u] = [B]^T[S][\delta_p] + [B]^T[S][\delta_0] \]

The solution is carried out in incremental form and Eqn 12 becomes
\[ [B]^T \left[ [S] \right] [\Delta u] = [B]^T \left[ [S] \right] [\Delta \delta_p] + [B]^T \left[ [S] \right] [\Delta \delta_0] \]
(13)

This is the basic set of equations for the problem but it is implemented in various ways depending on the state of the system. We are given initial conditions which typically involve zero displacements, axial forces, slip and temperatures. In this state a type A element is rigid. This implies that the change in the associated node displacement is zero and we set \( \Delta u = 0 \). The equilibrium equation for the node is taken out and subsequently gives the change in force in the type A bearing. When the type A bearing is rigid we monitor the forces in the bearing as \( \left[ \delta_0 \right] \) changes. We also monitor the forces in the type B bearings.

If we find \( N = \pm N_D \) in a type A bearing we modify the basic equations, now setting
\[ \Delta u = \Delta \delta_p \]
(19)

and \( S(\Delta u - \Delta \delta_p) = \pm N_D \)

Physically this is equivalent to replacing the bearing by an external force \( \pm N_D \) on the end node. We monitor \( \Delta u \) to ensure that \( \Delta u > 0 \) if \( N = +N_D \) and \( \Delta u < 0 \) if \( N = -N_D \). If this condition does not hold the bearing has unloaded and we again treat it as rigid.

If we find \( N = \pm N_S \) in a type B bearing we halt the temperature change and impose a change in the plastic slip
\[ [\delta_p] = \Delta \delta = (2N_S - N_D) \]
(15)

in the bearing in question. Changes \( [\Delta u], [\Delta \delta] \) are then calculated.

In the event that a type A bearing which is assumed to be plastic is found to have a jump in slip which is inconsistent with the force on the bearing as a result of slip at a type B bearing, we simply repeat the calculation now assuming that the type A bearing is elastic and will unload. We must also introduce a strategy to deal with the possibility that slip in a type B bearing can cause a type A bearing to yield or another type B bearing to slip. After the discontinuous changes \( [\Delta u], [\Delta \delta] \) are calculated we may find \( N/N_D \) in a type A bearing or \( N/N_S \) in a type B bearing.

If this occurs, additional plastic slips are introduced of such a magnitude to reduce the force on a type A bearing to \( N/N_D \) or of the conventional kind in a type B bearing. If yield occurs in both types of bearings, the type A bearings are treated first. This method of approach is not exact but the logical scheme proposed is not likely to lead to significant errors since the events are relatively uncommon.

Finally, we note that fixed abutments and fixed piers can be accommodated simply by increasing the \( N_D \) and \( N_S \) values respectively to such magnitudes that slip cannot occur.

Computer Implementation

A general purpose program for carrying through the analysis described in the previous section has been written. The program will analyse straight bridges of an arbitrary number of spans and an arbitrary history \( [\delta_0] \) may be imposed. Elastic stiffnesses of the deck elements and the piers may be arbitrarily chosen and the \( N_S \) and \( N_D \) values of each bearing can be individually chosen, with different values for slipping to the left and to the right if required.

In finite element terms the program is very simple; even on a long bridge the number of nodes will be very small. As a result it is feasible to formulate global matrices directly and find and store their inverses. In fact, four different stiffness matrices are formulated, corresponding to the state of the end abutment bearings (rigid-rigid, rigid-sliding, sliding-rigid and sliding-sliding). Various output options are available, the most important being a tabulation of internal forces and slips each time a slip occurs or a tabulation only at the peak values of temperature and at the zero crossings.

Examples

Details of two examples will be presented. The first is a simple, non-realistic model intended to illustrate the behaviour in a step by step fashion, while the second is the analysis of a real bridge.

The first example is shown in Fig 6. It is a symmetric continuous three-span steel bridge on end abutments and two piers. Each support has a PTFE bearing. The numerical details of the problem are given in the figure. The analysis has been carried out for a temperature variation which consists of a monotonic increase from 0°C to 16°C and a monotonic decrease back to zero. The main results are plotted in Fig 7: the quantities shown are the slips at the left hand abutment and at pier 1 and the interactive forces between the deck and the abutment and the deck and pier 1.
Following the temperature history, only the forces at the abutment change until the temperature $T = 1.16^\circ$C. At this point the abutments yield and slip begins. As the temperature is increased, slip proceeds at the abutments, with the force remaining constant, and the force on piers 1 and 2 increases. This latter force reaches the yield limit of 15 kN when $T = 6.94^\circ$C. Discontinuous slips occur, with the abutment slip increasing slightly and the pier forces dropping to 5.4 kN. Further increase in temperature again increases the force on the piers and discontinuous slip occurs again at 10.65°C. The process is again repeated, with the next slip occurring at 14.35°C.

When the temperature reaches 16°C and begins to decrease the abutments unload and become rigid. The force on the piers is then also constant, until the abutment bearings yield in the opposite direction when $T = 13.69^\circ$C. Further decrease in temperature takes place with the abutments sliding. Discontinuous slip occurs at the piers when $T = 4.17^\circ$C and $T = 0.46^\circ$C. The analysis was terminated when $T = 0^\circ$C.

The simple example shows clearly the essential features of the response, with the contributions of the rigid-plastic abutment bearings and the discontinuous sliding at piers and the saw-tooth response of the pier forces to monotonic temperature changes clearly seen.

The second example, shown in Fig 1 with the relevant data, is a real bridge which is continuous over seven spans. The piers are continuous with the deck (i.e., there is no sliding bearing at the top of the pier) at the top of piers 3, 4, and 5, whereas sliding bearings are present on the tops of other piers and the abutments.

In the particular calculation presented, the long-term effects of creep and shrinkage together with a temperature variation of 40°C were considered. Creep and shrinkage can be incorporated by introducing a fixed inelastic strain; this fixed inelastic strain can in turn be interpreted as a fixed temperature change. In this particular example creep and shrinkage strains were equivalent to setting the mean temperature at $-180^\circ$C. The temperature was thus assumed to vary between the limits of $+2^\circ$C and $-38^\circ$C.

Selected output from the program is shown in Fig 8. This includes the abutment and pier slips, the forces acting on two piers. Results are given only at the extremal values of temperature. Further detail cannot be practically shown on the diagram: in the half cycle $0^\circ$C to $2^\circ$C to $-38^\circ$C the computer carries out some 109 analyses, largely due to the small discontinuous slips (0.88 mm at a time) occurring in the stiff pier 1.

The results show that the response becomes periodic after the first time that the minimum temperature is achieved. It also shows that the piers "shake down" in the sense that no slip occurs at the pier tops once the solution is periodic. Periodic slipping does occur at the abutments, however.
Conclusions
The results obtained from the model and the calculation show broad general agreement with observed behaviour and give confidence that the model is at least reasonably reliable. The model can thus confidently be used as a design tool.
A number of interesting questions arise over the question of shake-down in all the bearings or, more realistically, in the type B bearings. It appears that very simple extensions to the classical shake-down-theorem can be given to cover the behaviour of type B bearings. This work will be reported in a further paper.

References

Discussion on papers
Written discussion on the papers in this issue will be accepted until 15 November 1981. This, together with the authors' reply, will be published in the May 1982 issue of The Civil Engineer in South Africa, or later.
Such written discussion, which must be submitted in duplicate, should be in the third person present tense, and should be typed in double spacing. It should be as short as possible and should not normally exceed 600 words in length. It should also conform to the requirements laid down in the 'Notes for the Guidance of Authors and Contributors' as published in the September 1974 issue of The Civil Engineer in South Africa.

Overseas contributors
For the convenience of overseas contributors only, the closing date for discussion will be extended to 30 November 1981 upon a receipt together with an assurance than the material will be received by the Institution by that date. No request for any further extension can be considered.

Reference
Whenever reference is made to above papers this publication should be referred to as The Civil Engineer in South Africa and the volume and date given thus: Ciev Engr SA Ir, Vol 23, No. 9, 1981.

Blockhouses during the SA war
D.P. DU PLESSIS

In the year 1901 during the South African War Lord Kitchener started to divide the whole country into compartments by constructing lines of blockhouses first along the railway lines and later also elsewhere. The purpose was to prevent the Boers from moving from one place to another and together with the space to the blockhouses lines. Essentially two types were built namely those which were substantial stone forts and the circular or octagonal corrugated sheet type (see photos). The stone forts had two or three storeys, were more expensive to build but were the most efficient type.
The corrugated sheet type had double walls with the space in between filled with gravel, earth or shingle. The earliest ones had two wooden frames to which each wall was nailed. It had a normal foundation and a gable roof. A later development had the space between the walls reduced to about 100 mm with only one wooden frame to which both walls were nailed. The normal foundation was replaced by an umbrella roof. The reduction in the wall thickness permitted a more efficient loophole constructed from sheet iron instead of cast iron in the shape of a double funnel with the neck in the middle of the wall.
Each blockhouse was furnished with a small cylindrical water tank of corrugated iron. It was in the making of these tanks, the iron of which had to be rolled in a machine to give the proper curve, that the idea of the circular blockhouse occurred. This proved to be the simplest and cheapest variety of all and became eventually the standard pattern. It consisted of two cylinders of corrugated iron without any woodwork, the annular space between the cylinders being packed with shingle and the whole roofed and loopholed as before. Blockhouses of this type were found to be more durable than those of the octagonal form which were liable to bulge. They were very cheap, required little material and little transport, could be rapidly turned out from central factories and placed in position with a minimum of skilled labour. As defensive works they were a success.

Acknowledgements
I wish to express my appreciation to Misses M. Olivier and S. Verwey of the Transvaal Archives Depot and the Union Buildings for providing the material on which this article is based.

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