\[ 2\tau_d + \tau_P = \gamma A_S \]

where \( \tau_d \) is the average shear stress on the physical wetted perimeter, \( P_w \).

Define \( k_c \) such that

\[ \tau_d (P_w + 2kd) = \gamma A_S \]

Combining these two equations gives:

\[ \frac{1}{k_c} = \frac{1}{\tau_d} \frac{A_p}{P_w} \frac{1}{d} - \frac{2}{P_w} \frac{d}{P_c} \]  
(14)

Similarly considering equilibrium of the flood plain:

\[ \tau_d + \gamma A_p S = \gamma P_p \]

and

\[ \tau_p (P_p - kd) = \gamma A_p S \]

Combining gives:

\[ \frac{1}{k_p} = \frac{1}{\tau_p} \frac{A_p}{P_p} \frac{1}{d} - \frac{d}{P_p} \]  
(15)

Having calculated \( k_c \) and \( k_p \) using Eqns 14 and 15, the modified wetted perimeters of the main channel and flood plain are calculated using Eqns 12 and 13, and the discharge of each subsection can be computed using a uniform flow resistance equation such as Manning's equation. Note that again the solution requires a knowledge of \( \tau_c \) for Eqns 14 and 15, hence an iterative solution using Eqn 10 or Eqn 11 (for example) is required as for the area method.

**Modified k method**

The \( k \) method can be simplified by defining the wetted perimeters as follows, assuming vertical interfaces and using an empirical result for \( k \).

\[ P_e = P_c + 2kd \]  
(16)

\[ P'_e = P_p \]  
(17)

Eqn 17 implies that the interface is always omitted from the wetted perimeter of the flood plain, which is a good assumption provided the flood plain is wide because the propelling effect of the main channel flow will then be small (James 16).

The difference in velocity between the main channel and the flood plain is responsible for the turbulence and shear stresses in the vicinity of the interface. Accordingly the ratio of the average velocity in the main channel to the average velocity in the flood plain region, \( v_v/v_{c} \), was used as the independent variable for defining an empirical relationship for the factor \( k \) in Eqn 16. This relationship was developed using the flume data of Wormleaton, Allen and Hadjipanoss 10 and that of Knight and Demetriou 13. A wide range of flow conditions are incorporated in these data, as can be seen in Table 1. Using these data, values of \( k \) are plotted as a function of \( vy_c/v_p \) in Fig 7. Although there is some scatter in this figure, a trend is clearly evident.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Data of Wormleaton et al 10</th>
<th>Data of Knight and Demetriou 13</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_p/W_c )</td>
<td>1.6</td>
<td>0.5; 1.0; 1.5</td>
</tr>
<tr>
<td>( n_p/n_c )</td>
<td>1; 1.27; 1.55; 1.91</td>
<td>1.0</td>
</tr>
<tr>
<td>( d/D )</td>
<td>0.11 - 0.43</td>
<td>0.11 - 0.51</td>
</tr>
<tr>
<td>( S )</td>
<td>0.00043; 0.00094; 0.00101; 0.00132</td>
<td>0.000966</td>
</tr>
<tr>
<td>( \tau_p )</td>
<td>0.58 - 17.4</td>
<td>0.30 - 28.1</td>
</tr>
<tr>
<td>No of data items</td>
<td>40</td>
<td>18</td>
</tr>
</tbody>
</table>

\( v_v/v_p \) in Fig 7. Although there is some scatter in this figure, a trend is clear and the following function was fitted by least squares regression, with a correlation coefficient of 0.77:

\[ \log k = 1.274 (v_v/v_p) - 2.235 \]  
(18)

The discharge in a symmetrical compound section can be calculated as follows using Manning's equation:

\[ Q = Q_c + 2Q_p \]

\[ = (A_p/\gamma R_v) R_v S_{ave}^2 + 2(A_p/\gamma R_p) R_p S_{ave}^2 \]  
(19)

where \( Q_c \) = flow rate in main channel, \( Q_p \) = flow rate in flood plain region, \( R_v = A_p/(P_w + 2kd) \) and \( R_p = A_p/P_p \).

**Fig 7: Relationship for the k factor in the modified k method**

For an asymmetrical section (one flood plain) the 2 is omitted from Eqn 19. Since \( k \) is a function of \( v_v/v_p \), an iterative solution is required, \( v_v/v_p \) is calculated using

\[ v_v/v_p = (n_p/n_c) (R_p/R_v) \]  
(20)

A value of \( v_v/v_p \) is assumed, \( k \) is calculated using Eqn 18, \( v_v/v_p \) is calculated from Eqn 20, \( k \) is re-calculated using this value of \( v_v/v_p \), and so on until \( k \) converges. Eq 19 is then used to calculate the discharge. The iterative procedure is shown in Fig 8. An accelerator is required for convergence. In Fig 8, \( i \) represents the current iteration and \( i-1 \) the previous iteration.

**Assessment of methods**

In order to assess the performance of the methods described above and to compare them with other methods, some of the flume data described above were used to compute discharges using the popular method, \( \lambda \) method, area method, \( k \) method and modified \( k \) method. The computed discharges were compared with the measured discharges and the difference expressed as a percentage error. The results are presented in Fig 9.

The general trend is that the error in computed discharges is greatest at lowest flow depths, which is when the turbulence intensity is at its highest. Comparing Figs 9(a) and 9(c) shows that the rougher the flood plain is relative to the main channel, the more the discharge is underestimated, because the velocity difference between the main channel and the flood plain is greater.
As can be expected, the popular method shows poor results, overestimating by up to almost 90 per cent. The λ method shows fairly good estimates of discharge except for the data set with a steep bed slope. James found that this method is not always accurate and can lead to main channel velocities being calculated smaller than flood plain velocities. The area method generally performs favourably compared with the other methods. Sometimes it overestimates the discharge, which could be attributed to Yen and Overton's belief that the friction factor needs to be modified as well. Good results could be obtained if empirical modifications to Manning's n or to the area adjustment C_A could be developed.

The k method performs better than the popular method but its overall performance is disappointing. This is probably because Manning's n needs to be modified to account for the fact that the resistance offered by the shear stress on the vertical interface is not the same as that posed by the bed roughness on the physical boundary. The modified k method shows very good performance under all test conditions, the error being within 10 per cent, which can be considered as the range of possible error in the measurement of the flood discharges and other parameters. When applying this method it must be borne in mind that it is applicable to wide flood plains, i.e. W_f /W_c > 1.0, the width of the flood plain being at least equal to the width of the main channel.

Conclusions

Five methods for predicting stage and discharge in compound channels have been assessed. These include the popular method and the λ method, which are probably the most widely accepted and practical techniques in current use, and three new approaches, viz the area method, the k method and the modified k method. Based on this assessment the following conclusions may be drawn:

1. The popular method grossly overestimates discharges at low flow depths and should not be seriously considered for discharge computation in compound channels.
2. The λ method is better than the popular method but has serious conceptual shortcomings.
3. The k method and the area method are potentially reliable and are conceptually more sound than the two existing approaches. Further research should be directed towards developing empirical modifications of these methods.
4. If the relative apparent shear stress η_c can be estimated (for example by using Eqs 10 and 11), the area method can be used in its present form as an improvement on both the popular method and the λ method.
5. The modified k method produces the best results and is recommended for general use. Refinement based on additional data for a wider range of conditions would be useful.

All the results presented in this paper are based on flume data, and experimental verification for large channels is still required.

References

3. Sellin, R H. A laboratory investigation into the interaction between the flow in the channel of a river and that over its flood plain. La Houille Blanche, Grenoble, France, No 7, 1964, pp 783 - 802.

List of symbols

\( A_c \) — cross-sectional area of main channel region, assuming vertical interfaces separating the main channel from the flood plains; \( A_c = W_c D \)
\( A_c' \) — modified cross-sectional area of main channel
\( A_f \) — cross-sectional area of flood plain region, assuming vertical
intersects separating the main channel from the flood plains;
\[ A_0 = W_d \]
\[ \Delta A \]
modified cross-sectional area of flood plain
area adjustment for the area method
\[ d \]
flow depth on flood plain
flow depth in main channel
\[ h \]
height of flood plain bed above main channel bed; \[ h = D - d \]
counter used in summations or iterative calculations
\[ i \]
modication factor for main channel wetted perimeter
\[ k_s \]
modification factor for main channel wetted perimeter
\[ k_f \]
modification factor for flood plain wetted perimeter
\[ n \]
Manning's roughness coefficient
\[ n_c \]
Manning's roughness coefficient for main channel
\[ n_e \]
equivalent Manning's roughness coefficient for compound section
\[ n_i \]
Manning's roughness coefficient for the i'th section of a compound channel
\[ n_p \]
Manning's roughness coefficient for flood plain
\[ P_s \]
wetted perimeter of the entire cross-section of a compound channel
\[ P_c \]
physical wetted perimeter of main channel; \[ P_c = W_c + 2h \]
\[ P_r \]
modified physical wetted perimeter of main channel
\[ P_l \]
wetted perimeter of the i'th section of a compound channel
\[ P_f \]
physical wetted perimeter of flood plain; \[ P_f = W_f + d \]
\[ Q \]
modified wetted perimeter of flood plain
\[ Q_c \]
discharge of compound channel
\[ Q_r \]
discharge of main channel region
\[ Q_l \]
discharge of one flood plain region
\[ R \]
hydraulic radius of compound channel
\[ R_c \]
hydraulic radius of main channel
\[ R_f \]
hydraulic radius of the i'th section of a compound channel
\[ R_s \]
hydraulic radius of flood plain
\[ S \]
longitudinal bed slope
\[ F_s \]
shear force on the physical wetted perimeter of the main channel
\[ F_p \]
shear force on the physical wetted perimeter of a flood plain
\[ \nu_c \]
average velocity in main channel region
\[ \nu_f \]
average velocity in flood plain region
\[ \Delta \nu \]
velocity difference between main channel and flood plain;
\[ \Delta \nu = \nu_c - \nu_f \]
\[ W_c \]
width of main channel
\[ W_f \]
width of flood plain
\[ \gamma \]
unit weight of water
\[ \gamma = \rho g \] where \( \rho \) is the density of water and \( g \) is the acceleration due to gravity
\[ \theta \]
angle of inclination to the horizontal of an inclined interface dividing the main channel and flood plain regions
\[ \lambda \]
apparent shear stress ratio used in the \( \lambda \) method for discharge calculation; \( \lambda \) is defined as the ratio of the apparent shear stress on a diagonal or horizontal interface, to the average shear stress in the main channel
\[ \tau_a \]
apparent shear stress on vertical interface between main channel and flood plain
\[ \tau_a \]
average shear stress acting on the physical wetted perimeter of the main channel
\[ \tau_f \]
average shear stress acting on the physical wetted perimeter of the flood plain
\[ \tau_r \]
relative apparent shear stress; \[ \tau_r = \frac{\tau}{\gamma d S} \]

Discussion on papers

Written discussion on the papers in this issue will be accepted until 31 October 1988. This, together with the authors' replies, will be published in the March 1989 issue of The Civil Engineer in South Africa, or later. For the convenience of overseas contributors only, the closing date for discussion will be extended to 30 November 1988. Discussion must be sent to the Secretaries of the SACE.

Whenever reference is made to the above papers this publication should be referred to as The Civil Engineer in South Africa and the volume and date given thus: Civ Engr S Afr, Vol 30, No 6, 1988.