Soil moisture flow theory

By O WIPPLINGER (Fellow)*

Synopsis
Vertical flow of soil moisture, in situations with water application, drainage and evapo-transpiration, is investigated through theory in which the dynamic effects of both water and air are taken into account. The hydro-pneumatic properties of two model soils - a fine-grained loam and a medium-grained sand - are derived from publications by investigators worldwide. A system of equations for defining soil characteristics is described, which can be applied to any soil tested in a specified manner. Lacking sophisticated laboratory service, complete data for a project soil, for which two specific readily determined properties are known, can be estimated from those of the model soils by interpolation. A method of analysis utilizing finite differencing is developed and its application demonstrated by numerical examples in aquifer recharge and irrigation. The complex analytical problems arising from the hysteresis phenomenon in matrix potential, in situations where there is cyclic change in soil moisture content, are solved.

Sameratting
Vertikale vloeistof van grondvloed, in omstandigheden met water toevoeging, dreining en evapotranspiratie, word deur teorie ondersoek, waarin die dinamiese uitwerking van beide water en lug in aanmerking geneem word. Die hydro-pneumatiske eienskappe van twee modelgronde - 'n fynkorreliere leem en 'n medium sand - word van wêreldwyw publikasies van wetenskaplike werkers afgeloop. 'n Stelsel vergelykinge vir grondkarakteristieke word beskryf, wat op enige grond wat op 'n voorgeskrewe wyse getoeis is, toegepas kan word. Waar 'n gesestikte laboratoriumdiens ontbreek, kan volledige gegevens vir 'n projekgrond, waarvoor twee spesifieke geredelik bepaalbare eienskappe bekend is, deur interpolasie van die van die modelgronde beraam word. 'n Ontledingsmetode waarin eindige elemente-berekenings aangewend word, word ontwikkel en die toepassing daarvan op numeriese voorbeelde in grondwateraanval en besproeiing gedemonstreer. Die ingewikkelde analytiese probleme wat voortspruit uit die histereseverkenning in matrikkansvoorde, in omstandighede waar daar sikliere verandering in grondvloed voorkom, word opgelos.

Introduction
Over the past 130 years soil moisture flow theory has been developed worldwide. Darcy¹ experimented with the flow of water in saturated sand, which led to Darcy's Law:

\[ q = -k \cdot \frac{dH}{dz} \]  

where \( q \) = the flux or rate of flow per unit of bulk area (measured at right angles to the direction of flow), m/s  
\( k \) = the coefficient of conductivity, m/s  
\( H \) = the hydraulic potential or the total hydraulic energy head as applied in Bernoulli's Law, m  
\( l \) = the distance in the direction of flow, m

The flow where only part of the voids were filled with water was much more difficult to analyse than that in saturated material, because in the former complicated matrix potential functions were components of \( H \) instead of the simpler hydraulic pressure terms in the latter. Although unsaturated soil moisture flow was physically very different from a diffusion phenomenon, it occurred in accordance with the following diffusivity equation in special non-hysteretic situations:

\[ q = D(s) \cdot \frac{ds}{dz} \]

where \( s \) = relative saturation or fraction of pore volume filled with water  
\( D(s) \) = diffusivity coefficient, m²/s (a function of \( s \))  
\( ds/dz \) = saturation gradient

Buckingham², after studying the movement of water in unsaturated porous media, extended Darcy's Law by introducing a saturation-dependent coefficient of conductivity \( k(s) \). By the chain rule of differential calculus, \( dh/ds = dh/dl \cdot dl/ds \). Therefore by Eqs 1 and 2 in the non-hysteretic situations referred to above,

\[ D(s) = k(s) \cdot \frac{dh}{ds} \]

Petroleum engineers applied Darcy's Law as extended by Buckingham, in the early 1920s, to analyse two-phase flow involving oil and gas or oil and water³. A concept of intrinsic permeability was introduced in terms of which, for a given porous medium saturated by any fluid, the quotient (conductivity)/(dynamic viscosity) was the same regardless of the fluid involved. In keeping with this, for air and water in soil, \( n_{w} \cdot k_{w} \) would be equal to \( 50k_{w} \), where \( n_{w} \), and \( k_{w} \) = Darcy's coefficients of conductivity in respect of air or water, determined when the porous medium conveyed only air or only water, respectively (m/s).

The factor 50 was approximately equal to the quotient (dynamic viscosity of air)/(dynamic viscosity of water).

The theoretical relation, \( n_{w} = 50k_{w} \), was found by laboratory experiments not to be strictly true, owing to 'gas slippage' or the Klinkenberg⁴ effect, after the investigator who first reported it. Brooks and Corey⁵ were amongst the first investigators to apply petroleum engineering principles to soil moisture flow. They found the above-mentioned constant to be 70,5 for Poudre River sand and 63,5 for Amarillo sandy loam. Following these results, in the interests of clear presentation of theory, did not use the intrinsic permeability concept, but developed my theory in a straightforward manner in terms of \( n(s) \) and \( k(s) \).

Brooks and Corey⁵ found non-dimensional curves for relative conductivities \( n/n_{w} \) and \( k/k_{w} \) for a wide range of materials (within which the model materials of the present investigation reside) displaying configurations differing little from material to material. These non-dimensional curves together with other available data yielded similar

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curves applied in the investigation (Fig 1).

Erickson solved a two-phase soil moisture flow problem, for which it was valid to apply diffusivity equations, using infinitesimal calculus and a Laplace transformation. Erickson's equations can be applied to simplify preliminary two-phase flow investigations or to study the effects of approximations made in solutions by finite difference analysis.

The relations between matrix potential and relative saturation were traditionally investigated by pressure and suction plate apparatus and also in special laboratories by means of more sophisticated equipment. In the latter, changes in moisture content in vertical soil-filled tubes of rectangular cross-section were monitored by gamma-ray attenuation, and matrix potential by sensitive tensiometers with membranes flush with the inner tube walls. Watson and Gillham were pioneers in research with the newer methods (see below).

**Hydro-pneumatic characteristics of porous media**

**Available model data**

Gillham determined the characteristics of an Eastern Colorado material known as gray dune sand in unusual detail. For matrix potential, he determined the two curves of the hysteresis loop and nine scan-

\[ \text{s, s}_{2}, \text{s}_{3}, \text{s}_{4} = \text{relative saturations (see Appendix)} \]

\[ k, n = \text{conductivities in respect of water and air, m/s} \]

\[ J, E, F, H = \text{data points for k} \]

\[ D, E, G = \text{data points for n} \]

\[ k_{m}, n_{D} = k \text{ and n at H and D respectively} \]

**Fig 1: Conductivities n and k for sand and for loam**

**Curve fitting:** Data by Watson, Brooks and Corey, Rubin, Morel-Seytoux and Klute et al. were used in the determination of the required two comprehensive soil characteristic models. In curve fitting, to derive functions from data, I applied rectangular hyperbola, of the general form \((s + A)(z + B) = C\), where \(A, B\) and \(C = \text{constants, s = relative saturation and z = m, or the low relative saturation part of a, or functions of m or k. Rectangular hyperbolae were adopted because the distribution of the plotted data, or functions of data, approximately matched that form.**

**Table 1: Co-ordinates of selected published data points used in deriving the functions k(s) and n(s) for Eqns 5 to 8 and Fig 1, converted to a non-dimensional form**

<table>
<thead>
<tr>
<th>Model material</th>
<th>Fluid</th>
<th>Point in Fig 1</th>
<th>Non-dimensional co-ordinates</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>s</td>
</tr>
<tr>
<td>Sand</td>
<td>Water</td>
<td>J</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>F</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>1.00</td>
</tr>
<tr>
<td>Air</td>
<td></td>
<td>D</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>0.95</td>
</tr>
<tr>
<td>Loam</td>
<td>Water</td>
<td>J</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>F</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td></td>
<td>H</td>
<td>1.00</td>
</tr>
<tr>
<td>Air</td>
<td></td>
<td>D</td>
<td>0.20</td>
</tr>
<tr>
<td></td>
<td></td>
<td>E</td>
<td>0.65</td>
</tr>
<tr>
<td></td>
<td></td>
<td>G</td>
<td>0.90</td>
</tr>
</tbody>
</table>

For the definition of Y see Fig 1

**Table 2: Co-ordinates of selected published data points used in deriving the hysteresis loop curve functions h(s) for Eqns 9 to 22 and Fig 2**

<table>
<thead>
<tr>
<th>Point</th>
<th>Model sand</th>
<th>Model loam</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>s</td>
<td>h</td>
</tr>
<tr>
<td>1</td>
<td>0.95</td>
<td>-0.34</td>
</tr>
<tr>
<td>2</td>
<td>0.525</td>
<td>-0.38</td>
</tr>
<tr>
<td>3</td>
<td>0.2875</td>
<td>-0.47</td>
</tr>
<tr>
<td>4</td>
<td>0.2</td>
<td>-0.6</td>
</tr>
<tr>
<td>5</td>
<td>0.05</td>
<td>-0.4</td>
</tr>
<tr>
<td>6</td>
<td>0.95</td>
<td>-0.19</td>
</tr>
<tr>
<td>7</td>
<td>0.525</td>
<td>-0.25</td>
</tr>
<tr>
<td>8</td>
<td>0.8275</td>
<td>-0.35</td>
</tr>
<tr>
<td>9</td>
<td>0.9375</td>
<td>-0.17</td>
</tr>
<tr>
<td>10</td>
<td>0.9375</td>
<td>-0.095</td>
</tr>
<tr>
<td>11</td>
<td>0.95</td>
<td>0</td>
</tr>
</tbody>
</table>
The control points for curve fitting (Tables 1 and 2) were selected by inspection, from plotted data. In the hysteresis diagram three control points were used for each of the curves, \( m, \sigma \), and the low relative saturation part of \( a \). Other curves shown in Fig 2 were derived from these curves, for example for sand, by Eqsns 13, 14 and 16 below. Four and three control points respectively were used for the conductivity curves in respect of water and air. Greater refinement in the derivation of functions was not considered justified with the information available.

Rubin\(^{17}\) estimated the co-ordinates at the low relative saturation end of the hysteresis loop, from observations of humidity of soil air, and soil moisture content in a stockpile of Rehovoth sand, to be \((-60 m, 0.05)\) and \((-100 m, 0.2)\) respectively (Table 2).

Scanning curve functions were derived from a primary drying scanning curve selected from the curves published by Watson\(^6\) and shown schematically as DP4 in Fig 2. The point \( P \) on this curve was situated half-way between the two curves of the hysteresis loop on a line of constant saturation. In Fig 2

\[
Z = (s_0 - s_h)
\]  

Rubin\(^{17}\) had used functions for primary drying scanning curves containing a factor \( e^{e^{\text{Cef}-\text{C}}} \) where \( e \) was the base of Napierian logarithms, \( C \) a constant for the material, and the symbols \( s_0 \) and \( s_h \) the relative saturations at two points on the curve. \( D_R \) was the point of reversal from wetting to drying from which the primary drying scanning curve had originated on a wetting loop curve. I used a somewhat different factor, \( e^{e^{Cef}-\text{C}} \), in an equation relating a present to a previous matrix potential at any two points on a curve of monotonic change in relative saturation. Here, \( s \) and \( y \) are present and previous relative saturations, and \( C \) is a constant.

With the new factor \( e^{e^{Cef}-\text{C}} \) applied in Eqn 9 or 10 below, matrix potentials \( h \) on scanning curves of any order (primary, secondary, tertiary, etc) were calculated step by step for given changes in \( s \). Reversals from drying to wetting and vice versa were assumed to occur at the ends of step intervals. Whenever monotonic change of sufficient duration occurred, scanning curves approached the loop curves very closely, with complete equality wherever the end points of the loop were reached (Fig 2).

### Characteristics

1. Conductivity: Fig 1 expresses \( n \) and \( k_h \), in terms of \( n_0 \) and \( k_{0_h} \), where

\[
n_0 = n_0 + n_{0_h} \text{ and } k_h = k_{h + k_{0}}
\]

\( n_0 \) at \( s = s_h \) replaces \( n_{0_h} \) at \( s = 0 \) in accordance with a simplification used by Brooks and Corey:\(^7\)

The following extreme values for \( n \) and \( k \) were estimated from available data and applied in Table 1:

- **Sand**
  - \( n_0 = 0.023 \text{ m/s} \)
  - \( k_h = 3.20 \times 10^{-10} \text{ m/s} \)

- **Loam**
  - \( n_0 = 2.00 \times 10^{-10} \text{ m/s} \)
  - \( k_h = 2.80 \times 10^{-10} \text{ m/s} \)

The following equations were derived by curve fitting procedures using the data in Table 1:

- **Sand**
  - \( k = 1.084 \times 10^{-10} \text{ [-1 - 2.418(s - 2.468)]^{2.8}} \)  
  - \( n = 1.284 \text{ [1 + 2.868(s - 3.635)]^{2.8}} \)  
  - **Loam**
  - \( k = 1.572 \times 10^{-10} \text{ [-1 - 2.281(s - 2.481)]^{2.8}} \)  
  - \( n = 4.29 \times 10^{-10} \text{ [1 + 1.392/(s - 2.292)]^{2.8}} \)

2. Matrix potential: For both sand and loam

\[
\text{If } s > y \quad h = b + \frac{(s - w)(a - b)}{(w - w)} e^{e^{\text{Cef}-\text{C}}} \\
\text{If } s < y \quad h = a + \frac{(g - u)(a - b)}{(u - w)} e^{e^{\text{Cef}-\text{C}}} \\
\text{If } s = y \quad h = g
\]

For sand \( s > or < 0.2875 \)

\[
m = -0.297 - 0.037871/(s - 0.06859) \quad (11) \\
\sigma = -0.1093 - 0.080437/(s + 0.04668) \quad (12) \\
a = m[1 - e^{0.56(0.999)}] \quad (13) \\
b = a[1 - e^{0.56(0.999)}] \quad (14)
\]

For sand \( s < 0.2875 \)

\[
a = -0.2451 - 0.053709/(s - 0.048649) \quad (15) \\
b = (3985/3953)(40 + a) - 40 \quad (16)
\]

For loam \( s > 0.4 \)

\[
m = -0.3987 - 0.185273/(s - 0.2639) \quad (17) \\
\sigma = 0.1925 - 0.179455/(s - 0.2866) \quad (18) \\
a = m[1 - e^{0.56(0.999)}] \quad (19) \\
b = a[1 - e^{0.56(0.999)}] \quad (20)
\]

For loam \( s < 0.4 \)

\[
a = 0.4176 - 0.445151/(s - 0.195567) \quad (21) \\
b = (9861/9824)(100 + a) - 100 \quad (22)
\]

All the variables in Eqsns 9 to 22, apart from the relative saturations \( y \) and \( s \) at the beginning and end of a step in computation, were matrix potentials defined in detail in Table 3.

### Table 3: Notation: Eqsns 9 to 22

<table>
<thead>
<tr>
<th>Curves with matrix potential ordinates</th>
<th>Matrix potentials, ((m))</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Asymptotes</strong></td>
<td><strong>Given at beginning of a step</strong></td>
</tr>
<tr>
<td>Loops curves</td>
<td><strong>Calculated at end of a step</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Drying</strong></td>
</tr>
<tr>
<td>Scanning curves</td>
<td>( u )</td>
</tr>
</tbody>
</table>

Only one value of \( Z \) was chosen for a particular material, because of a measure of symmetry apparently correlating the drying and wetting scanning curves, and in order to simplify computation by not varying \( Z \) with relative saturations. Values of \( Z = 0.077 \) and 0.05 were derived from selected primary drying scanning curves in hysteresis diagrams published by Watson\(^6\) for sand and loam. \( C \) was evaluated as follows.

The matrix potentials at the current point \( P \) and the preceding point \( D \) on curve DP4 in Fig 2 were \( h \) and \( g \) respectively. Because of the mid-position of \( P \), and the position of \( D \) on the wetting curve of the loop,

\[
h - a = (b - a)/2 \\
g = w
\]

(23)

Substituting Eqn 23 in Eqn 10, and by Eqn 4 with \( s_h = y \) and \( s_0 = s \),

\[
(b - a)/2 = (b - a)e^{e^{Cef}} \\
\therefore e^{e^{Cef}} = 2
\]

(24)

From Eqn 24 and the values \( Z = 0.077 \) and 0.05 given above:

For the model sand: \( C = 9 \) For the model loam: \( C = 14 \)

It was a special property of the scanning curve generating function \( e^{e^{Cef}-\text{C}} \) that \( g - y \) could have any value between 0 and \( s_h - s_0 \) with the starting point at relative saturation \( y \) situated anywhere on a scanning curve of any order, provided the end point at saturation \( s \) was on the same curve (monotonic change). \( s_0 \) and \( s_h \) were approximate concepts ignored by some investigators. Their value could differ in the same field or laboratory, for instance, with differences in time since water application. In this investigation they were presumed to be constant as listed for points 5 and 11 in Table 2.

3. Estimates of \( \phi \) and \( s_r \)

Model sand: \( \phi = 0.35 \) \( s_r = 0.85 \)

Model loam: \( \phi = 0.475 \) \( s_r = 0.80 \)

where \( s_r \) is the relative saturation at the interface between the capillary fringe and the zone of lesser water content and \( \phi \) = porosity of the material.
Derivation of characteristics by interpolation: Where an adequate laboratory is not available, a complete set of characteristic functions for a project soil can be estimated, if the following laboratory determinations are available:

- \( k_s = k \) of the project soil when \( s = 1 \), corresponding to \( k_{UU} \) and \( k_{BU} \) defined in the same way for the model materials.
- \( s_l \) = s of the project soil, when \( h = -1 \) m in a drying test commencing at \( s = s_0 \) corresponding to \( s_{ML} \) and \( s_{MU} \) found in the same way for the model materials.

By applying the proportionalties \( k_{ML} : k : k_{MU} \) and \( s_{MU} : s : s_{MU} \), the coordinates of the control points, listed in Tables 1 and 2 for the model soils, can be estimated for the project soil.

Flow analysis

Basic assumptions

Moisture flow was assumed to occur in the liquid state. In fallow land and aquifers during the dry season, vapour transfer will reduce soil moisture storage, a long-term effect requiring field observations and theory not considered in this investigation. Assumptions made in this analysis were:

1. At relative saturations less than those in the capillary fringe, water and air occupy separate micro-channels, the outer walls of air channels being the undulating surface of the menisci of the adjoining water phase.
2. An increase of water content from the situation in 1 to that in the capillary fringe was accompanied by a collapse of the air channels, with air thereafter occupying discontinuous irregularly-shaped bubbles surrounded by water, acted upon by forces due to upward buoyancy and resisting surface tension.
3. At \( s = s_o \), a residual portion of the air referred to in 2 above was firmly held to adjoining solid particles, so that it in effect became part of the solid matrix.

The movement of air in 1 is brought about by air pressure gradients in accordance with Darcy's Law. In 2 this is not the case, owing to the discontinuity of the air bubbles. The result is an air entry pressure phenomenon at the menisci enclosing the capillary fringe, the implications of which are described later. The water phase is continuous throughout 1 and 2 with flow determined by gradients of hydraulic potential and Darcy’s Law at all levels.

Within a micro-tube dipped into water, \( s = 1 \) and \( h = 0 \) anywhere within the capillary rise. In the capillary fringe above a water table in a granular material, however, both \( s \) and \( h \) decrease with increasing height, as proved by experiments by Klute et al.\(^{14,15} \). Estimated allowance was made for this phenomenon to complete Watson’s diagram above the apparent truncation level referred to before. The thus amended diagram was the basis for the present hysteresis model, Fig 2, in which \( ds/dh \) is never zero over the full range \( s_a < s < s_o \).

Conceptual vertical column

In the following analysis an enclosed vertical column of the porous material is considered, subject to conditions of influx and efflux at the upper and lower boundaries. The upper boundary was at the surface of the porous medium and the lower boundary at a constant depth, greater than the depth of the water table. The notation used in this regard is given in Fig 3.

Particulars of profiles \( s(x) \) and \( h(x) \) and boundary conditions \( q_U \) and \( q_L \) were required as input data in the first interval of the finite difference analysis. In every subsequent interval, the initial conditions were the computed preceding values for \( s(x) \), together with \( h(x) \) derived from \( s(x) \) and \( y(x) \) by Eqs 9 and 10, and the given or computed \( q_U \) and \( q_L \). Values of \( k(x) \) and \( q(x) \), derived from ordinates of the profiles \( s(x) \) and Eqs 5 to 8, were then applied together with the ordinates of the profiles \( h(x) \) in the derivation of the equations for the interval-average flow rates \( q \) at depths \( x \) in the profile. The parameters \( q \) in the analysis were thus based on the initial conditions of the interval only, a permissible approximation for sufficiently short time intervals.

The intervals \( t = 0.01 \) and 0.05 hours used here were appropriate for the associated respective rates of intake, reaching maxima of 900 mm/ hour and possibly 100 mm/hour in aquifer recharge and during rain storms in irrigation respectively. The initial \( s(x) \), \( h(x) \), \( q_U \), and \( q_L \) together with the equations for interval-averaged flow rates at depths concerned were applied to determine a matrix of equations for the derivation of a new \( s(x) \) as an initial profile for the next interval.

The dynamics of unsaturated flow in micro-conduits

Derivation of partial differential equations: Although the zones of different relative saturation in the column (with \( s_a < s < s_o \) and \( s = s_r \) respectively) were to be analysed as a single system, it was convenient to consider the range \( s_a < s < s_r \) first. In addition to the previous principles, an assumption made in two-phase petroleum flow theory was applied, that the undulating surface of the wetting fluid separating the two phases was either a stationary common conduit wall or moved slowly enough not to affect air flow.

Conversely, air flow did not significantly affect the movement of the conduit wall. Apart from the reduction in air volume brought about by ponding or incipient ponding in runoff-producing sprinkler irrigation or rain, air was assumed to be incompressible, air pressure differences due to Darcian flow being too small to cause significant volumetric changes.

For the range \( s_a < s < s_r \) and no volumetric change in soil air:

\[
q_U + q_{BU} = q + q_w = q_L
\]

(25)

By volumetric continuity of uncompressed air, and because \( s_a = 1 - s \), where \( s_a = \) fraction of void space filled with air,

\[
\frac{\delta q_a}{\delta x} = \frac{\delta s}{\delta t}
\]

(26)

The continuity equation for water:

\[
\frac{\delta q_w}{\delta x} = -\frac{\delta s_a}{\delta t}
\]

(27)

As the velocity, \( V \), in Darcian flow was small, \( V^2 \) in Bernoulli’s Law for hydraulic potential, \( H \), was negligible, yielding the simple form: \( H = P_a \), where \( P_a = \) water pressure in metres head of water.

\[
\frac{\delta H}{\delta x} = \frac{\delta P_a}{\delta x} - 1
\]

(28)

Darcy’s Law, Eqn 1, as extended for unsaturated conditions, and Eqn 28 yield

\[
q = -k \frac{\delta H}{\delta x} = k \left( 1 - \frac{\delta P_a}{\delta x} \right)
\]

(29)

From Darcy’s Law applied to the flow of air
\[ q_b = -n \frac{\delta P_b}{\delta x} \]  

Multiplying Eqn 29 by \( n/k \)

\[ n \frac{k}{k} q = n - n \frac{\delta P_a}{\delta x} \]  

Eqn 31 – Eqn 30

\[ \frac{n}{k} q - q_e = n \left( 1 - \frac{\delta P_a}{\delta x} \right) \]  

From equilibrium of pressure in the water,

\[ h = P_a - P_b \]  

From Eqns 32 and 33, applying the chain rule of differential calculus,

\[ \frac{n}{k} q - q_e = n \left( 1 - \frac{\delta h}{\delta s} \right) = \left( 1 - \frac{\delta h}{\delta s} \frac{\delta s}{\delta x} \right) \]  

It should be noted that \( \frac{\delta h}{\delta s} \) in Eqn 34, a partial differential coefficient derived from two known profile relations \( s(x) \) and \( h(x) \) in hysteresis-affected situations, is very different from \( \frac{dh}{ds} \) derived by differentiating the hysteretic \( h(s) \) relation of the moist material. The two are identical in special non-hysteric situations where \( \frac{dh}{ds} \) can be used. Eqn 34 with \( \frac{dh}{ds} \) is correct in all situations.

Eliminating \( q_e \) from Eqns 25 and 34

\[ q = \frac{(n + q_e)}{(1 + n/k)} - \frac{\delta h}{\delta s} n/(1 + n/k) \]  

Parameters \( P_a, \) (m/s) and \( r, (m^2/s) \) were introduced to simplify presentation

\[ p = \frac{(n + q_e)}{(1 + n/k)} = k \left( \frac{\delta h}{\delta s} \right) n/(1 + k/n) \]  

From Eqns 35, 36 and 37

\[ q = p - r \frac{\delta s}{\delta x} \]  

From Eqn 27

\[ \phi \frac{\delta s}{\delta t} = - \frac{\delta q}{\delta x} \]  

From Eqns 38 and 39

\[ \phi \frac{\delta s}{\delta t} = \delta \left( \frac{\delta s}{\delta x} - \frac{\delta P}{\delta x} \right) \]  

From Eqn 38

\[ \frac{\delta s}{\delta x} = \frac{(p - q)r}{r} \]  

System of finite differencing: Additional notation is as follows:

\[ \nu \]  

the given relative saturation at the beginning of \( \delta t \)

\[ d \]  

the unknown relative saturation at the end of \( \delta t \)(\( d \) computed in one \( \delta t \) became the known \( \nu \) of the next \( \delta t \))

\[ j \]  

a subscript indicating depth below surface (at the top of the column \( j = 0 \), at the bottom \( 80 \))

\[ \Delta x \]  

an advance in depth equal to \( 2j (\Delta x = 0.125 m) \); at the top of the column \( x = 0 \), at the bottom \( m \)

Eqn 40 was applied to derive 40 matrix equations, one at each of the odd-numbered nodes \( j = 1 \) to 79. In the derivation of the two equations at the extreme nodes \( j = 1 \) and \( j = 79 \), Eqn 41 partly replaced Eqn 40, inter alia to introduce boundary conditions. Eqn 40 was expressed as follows in finite difference form for the 38 nodes \( j = 3 \) to 77:

\[ \phi \Delta s = \left( \Delta s \right)_{j+1} - \left( \Delta s \right)_{j} \]  

\[ (d - \nu)/\delta t \]  

For even-numbered nodes between \( j = 1 \) to 79, the following equivalents for \( \frac{\delta s}{\delta x} \) and \( \frac{\delta h}{\delta x} \) were derived by linear interpolation:

\[ \left( \frac{\delta s}{\delta x} \right)_{j+1} = \frac{\delta s}{\delta x} \]  

\[ \left( \frac{\delta h}{\delta x} \right)_{j+1} = \frac{\delta h}{\delta x} \]  

In the foregoing equations \( d_{j+1}, d_{j+2}, d_{j+2}, \) and \( d_{j+2} \) were the dependent variables. By separating dependent from independent variables a tridiagonal matrix of 40 finite difference equations was derived from Eqn 42 after making the necessary substitutions in terms of Eqns 43 to 47, and by applying Eqn 41 at the boundaries. The three standard equations below served as notation for all the terms in the above-mentioned 40 equations:

\[ B_d d_1 + C_d d_2 = D_1 \]  

\[ A_1 d_{j+1} + B_1 d_j + C_1 d_{j+2} = D_j \]  

\[ A_{26} d_7 + B_{26} d_6 + C_{26} d_5 = D_{26} \]  

The index \( j \) in Eqn 49 referred to odd numbers from 3 to 77 only. The terms \( A, B \) and \( C \) were matrix coefficients, and \( D \) the so-called vector terms. The \( \Delta x \) was linear, ie the equations were linear in \( d \), and the coefficients and vector terms, being functions of initial conditions of intervals to which the matrices applied, were constants. These functions together with two constants \( G_1 \) and \( G_2 \) used therein were as set out below:

\[ G_1 = 2b(\Delta x)^2/\delta t \]  

\[ G_2 = 2G \]  

\[ A_1 = 0 \]  

\[ B_1 = G_1 + r_2 \]  

\[ C_1 = -r_2 \]  

\[ D_1 = G_1 v_1 - r_2 (v_1 - v_j) - G_2 (d_2 - q_2) \]  

\[ A_{11} = -r_{j-1} \]  

\[ B_{j} = G_1 + r_{j+1} + r_{j-1} \]  

\[ C_{j} = -r_{j-1} \]  

\[ D_{j} = r_j v_{j-2} + (G_1 - r_{j}, r_{j+1}, v_{j+1}, v_{j+2} - G_2 (d_{j+1} - d_j)) \]  

\[ A_{26} = -r_{7} \]  

\[ B_{26} = G_1 + r_{7} \]  

\[ C_{7} = 0 \]  

\[ D_{26} = G_1 v_{7} - r_{7} (v_{7} - v_{j}) + G_2 (d_{7} - q_{7}) \]  

The stability of matrix solutions: Cumulative computer errors developed in the repeated solution of the 40 x 4 matrices of simultaneous equations. The following adjustments were introduced to stabilize the results.

1. Whenever computed \( h(x, s) \) co-ordinates of one interval and therefore \( q(y, x) \) of the next, in conflict with the basis of analysis, represented points outside the hysteresis loop, unstable profiles \( h(x) \) and \( s(x) \) developed. The instabilities were completely eliminated by introducing two conditional command equations in the computer program:

\[ h_j < a_j, h_j = a_j \]  

\[ h_j > b_j, h_j = b_j \]
2. As a result of finite differences being applied in the analysis to simulate true situations with infinitesimal increments, the computed profiles tended to include s-ordinates greater than $s_0$ and smaller than $s_p$. Such values were beyond the range for which Eqs 5 to 22 were valid and if uncorrected would have resulted in great deviations. Different procedures were adopted for corrections relating to $s_0$ or $s_p$, each of which satisfied the water balance requirements.

3. Drying profiles sometimes show a distance below the point of inflexion in an advancing soil moisture front changed to a state of wetting in terms of scanning curve relations, at first with very small increments of $s$. In this initial stage of transition the factor $1/(s/s_2 - s)$, occurring in $r_{a+1}$, and therefore in matrix coefficients, changed from positive values through $+\infty$ to $-\infty$ and finally to less extreme negative values. There was a single-phase solution for $s_{i+1}$, but values of $1/(s_i/s_2 - s)$ approaching $+\infty$ (which usually did occur in a few of the 480 or 2400 time intervals during a day for $t = 0.05$ or 0.01 respectively) gave rise to unstable matrixes and erroneous solutions. Stability was ensured by equating values of $1/(s_i/s_2 - s)$ between $-\infty$ and $-10^4$ to $-10^6$, and those between $+\infty$ and $10^4$ to $10^6$.

4. Residual rounding-off errors occurred in the ordinates of the relative saturation profiles. Errors in the sixth decimal place of the mean profile value were eliminated by an evenly distributed compensation over the profile in each time interval, as a final step in stabilization.

Four zones analysed as a single system

The following four zones occurred in the conceptual column from the upper boundary downwards.

1. Zone UC = the upper capillary zone occurring with ponding or runoff-producing irrigation or rain, where a saturated state was assumed at the upper boundary and $s = s_p$ at all other depth indexes $j$ in the zone.

2. Zone MC = that part of the unsaturated zone in which $s$ was variable and less than $s_p$, and in which air was assumed to flow in micro-conducts. This zone extended from the lowest level of zone UC downwards.

3. Zone LC = the lower capillary fringe extending from the water table upwards to an interface with the zone MC at a level where $s = s_0$.

4. Zone SA = the saturated zone below the water table in which $s = s_0$.

In terms of the physical concepts adopted herein, there were no micro-conducts in zones UC and LC. The air in excess of the irreducible fraction 1 - $s_p$, which did occur in these zones, in terms of Fig 2, was assumed to be present in irregularly shaped bubbles loosely held between the solid particles of the porous medium. The relative saturation in zone LC varied between $s_i$ and $s_0$. The excess air in this situation moved easily by buoyancy, and not by Darcian flow of air in micro-conducts.

The idea that there was in fact a fundamental change in the mode of air movement at $s = s_p$ was confirmed by trial computations on the assumption that micro-conducts of ever-decreasing size down to zero cross-section (when $s$ had increased to $s_p$) could exist and that Darcian flow continued according to the air conductivity curves of Fig 1. The unrealistically high pressures computed as a result proved that air was not present in micro-conducts in zone LC. This obviously applied to zone UC as well.

The system of finite differencing, with its stabilization routines, was applied in the analysis of the multi-zone column for single-phase and two-phase flow. Particulars were as follows:

**Flow in zone UC**: By definition UC exists only if the soil at the upper boundary is saturated, so that $h = 0$ at $j = 0$. Air compression in MC commences when $s$ at $j = 1$ has increased to $s_0$. With continued ponding or incipient ponding a wetting front with $s = s_p$ is assumed, proceeding downwards from $j = 1$, with a column of soil moisture in its wake in which $s = s_p$.

**Additional notation**:

- $\rho_o = \text{depth of ponding in metres}$
- $\rho_o = \text{absolute atmospheric pressure, eg 10 m head of water}$
- $\rho_o = \text{absolute current upward air pressure on the lowest level of zone UC in metres head of water}$
- $\rho_r = \text{the value of } \rho_o \text{ one time interval before current time}$
- $k_p$, $k_s$, and $h_r = s$, $k$, and $h$ in zone UC
- $m_s$ and $m_r = \text{current and preceding mean s-ordinate of the profile respectively}$
- $q_o$ and $q_r = \text{rate of water application and intake respectively}$
- $j_f = \text{depth index at the lowest level of zone UC}$
- $\Delta p = \text{resultant upward external pressure, resisting downwar gravity and matrix potential forces, exerted on the column of water in zone UC}$
- $h_r = \text{the matrix potential of the water at the wetting front}$
- $b_{w_0} = \text{(ordinate on wetting loop curve)}$

By definition: $\Delta p = \rho_o - \rho_r - \rho_0$ (67)

Air is compressed progressively until $\rho_p$ reaches a maximum at which $\Delta p = -h_r = \text{the air entry value of the soil, with air subsequently bubbling up through UC at the same rate as water flows downwards}$. $\rho_p$ is estimated by Boyle's Law by the following equation:

$\rho_p = \rho_o(1 - m_r)/(1 - m_s)$ (68)

Intake rates were estimated by considering pressures and matrix potentials acting on the column of soil moisture in UC in which the coefficient of conductivity was assumed to be $k_p$. Three conditional equations were derived.

If $s_{i+1} > s_0$, $q_r = k_i(1 + (\rho_r - h_{i+1}))(0.5 \Delta x)$ (69)

If $s_{i+1} > s_0$, $q_r = k_i(1 - (-\Delta p - h_i))(0.5 j_i \Delta x)$ (70)

If $\Delta p = -h_r$, $q_r = k_i$ (71)

An approximation was made in the above by assuming that in the early stage of the existence of UC, while Eqs 69 and 70 were applicable, $s = s_p$ at all levels down to $j_f = j_r$. In the real situation $s > s_p$ initially occurred near the upper boundary, changing to a uniform distribution with $s = s_p$ when $\Delta p = -h_r$ was reached. The assumption led to a conservative estimate of intake rates in the initial period with steep hydraulic gradients.

**Flow in zones MC, LC and SA**: The computer program applies finite differencing in terms of Eqs 42 to 66, without amendment, in zone MC and, with air conductivity terms omitted, in zones LC and SA. The program computes continuous profiles $s(x)$ and $h(x)$ in one integrated process for the four zones of the conceptual column.

**Single-phase flow in the column as a whole**: If the soil air was considered to be free to escape laterally, and water to move vertically by laminar flow under the combined influence of gravity and vertical matrix potential forces, computer rates of intake and moisture front advance were the same approximated in real situations with small or elongated areas of water application. The intake rate equations in this theoretical single-phase flow situation were

If $s_{i+1} > s_0$, $q_r = k_i(1 + (\rho_o - h_{i+1}))(0.5 \Delta x)$ (72)

If $s_{i+1} > s_0$, $q_r = k_i(1 + (\rho_o)(0.5 j_i \Delta x)$ (73)

where $j_i = j$-index at a presumed wetting front with $s = s_0$.

**Non-laminar flow**: Allowance for the change in mode of flow when eddy currents developed and the limits of laminar flow were exceeded was made by assuming that $q_r$ would not exceed an arbitrary ceiling value of $b_k$, and that the restrained flow would be laminar. This arbitrary approximation was made to simplify the simulation of a complex situation of mode flow change at great hydraulic gradients. Errors will be limited owing to the extremely short duration involved at the commencement of ponding or runoff-producing water application.

**Regional water table fluctuations below irrigated land**: If $q$, is assumed to be zero at the base of the conceptual column, the water table would be computed to rise owing to net inflow = irrigation + rain intake - evapo-transpiration. At a selected time, preferably annually at the end of the dry season, the computer water table level and observation well levels will be found to differ owing to groundwater flow into and out of the aquifer beneath the area irrigated. Corrective positive or negative
q_i values will have to be calculated and applied in ensuing years to maintain approximate long-term agreement of computed and observed water table levels.

**Pipe drains below irrigated land:** Conceptual column depth is chosen equal to that of the pipe drains, with assumed water table at a constant level maintained in pump sumps on the collecting drains. In the simulation, rates of efflux q_i will be determined in each interval \( \Delta t \) to maintain the constant water table. Where drains are only partly within zone SA, or discharge freely to open channels at lower levels, two-phase flow effects and air compression due to ponding will become negligible, q_{int} will not be zero as assumed in Eqn 25, and the assumption of single-phase flow will be appropriate.

**The computer program and its application**

A computer program for simulating vertical flow in uniform columns of soil was derived from the foregoing theory. Its validity was proved solving a non-hysteretic two-phase problem (Erick's), by the close agreement between the results it yielded and an accurate solution by Erick's infinitesimal calculus method. In hysteresis-afflicted problems in groundwater recharge and irrigation the suitability of the adopted \( \Delta t \) was proved by the close agreement found in test runs with 0.5\( \Delta t \) and 2\( \Delta t \). Application of scientific analysis based on fluid mechanics contributed to confidence in the correctness of the results obtained by applying the program.

**Sand-aquifer recharge**

An aquifer 5 m deep above an impervious base was considered, with a given initial soil moisture profile \( s(x) \), such that the vertical flux at all depths down to \( j = j_{w} \) was approximately zero, in terms of drying relations of the hysteresis diagram and Eqn 29, where \( j_{w} \) = the \( j \)-index at the water table.

Ponding one metre deep was assumed, abruptly commencing at \( t = 0.02 \) hours and terminating at \( t = 1.02 \) hours. \( \Delta t = 0.01 \) hour was chosen for the computation. Computer runs were performed for two problems with two-phase and single-phase flow respectively, and no evapo-transpiration or pumped extraction. Computed profile evolution and intake rates were as shown in Figs 4 and 5. The initial \( s(x) \) profiles in both instances were derived by substituting \( m \) or \( a = - \)depth above \( j_{w} = 71 \), in Eqns 11 or 15, for \( s > = 0.2875 \) or \( s < 0.2875 \) respectively. This initial condition, within a few time intervals, led to a stable profile with \( j_{w} = 69 \).

**Irrigation**

Simulation by computer program: The program was applied with the following inputs to simulate soil moisture regimes in the irrigation of vegetable crops grown on the model sand:

1. Depth of rooting zone = 1 m.
2. Distribution of evapo-transpiration rate during the day, in simulations where only the daily amount was given:
   a. None between 6 pm and 6 am.
   b. Rate proportional to the height above the base of the rooting zone and to the fraction of six hours elapsed after 6 am in the forenoon or still to elapse up to 6 pm in the afternoon, with a peak value at noon.
3. The initial \( s(j) \)-profile in simulations was derived as described in the aquifer recharge problems, with an amendment so that \( s(j) \) would nowhere be less than the value of \( s \) when \( a(s) = -1 \) m (nominal field capacity profile).

**The Philip farmer’s irrigation practice:** J Gorgens16, a farmer irrigating sand similar to the model sand of this investigation in the sand flats area at Philip near Cape Town, developed a successful irrigation practice for his vegetable crops, in which water was applied at the rate of 5 mm/h average over area for 2.5 hours each day during the period of maximum consumptive use of the crops. This practice was investigated by computing the soil moisture and matrix potential in the rooting zone, with inputs 1 to 3 as listed above and the following additional data:

- \( N_{a} = \) the period between water applications = 1 day
- \( C_{a} = \) an assumed distribution factor = 0.75

**Fig 4: Two-phase flow. Sand aquifer recharge**

![Fig 4: Two-phase flow. Sand aquifer recharge](attachment:image)

**Fig 5: Single-phase flow. Sand aquifer recharge**

![Fig 5: Single-phase flow. Sand aquifer recharge](attachment:image)

**Formulan in conventional scheduling are as follows:**

\[ I = C_{a}I_{S} \] (74)

\[ I_{a} = C_{a}I_{S} \] (75)

Here, \( I \) and \( I_{a} \) (mm) = scheduled areal mean and corresponding least favoured part applications respectively.

Simulation by Eqn 74 in a 20-day run with \( e_{S} = 7.5 \) mm on each day and 45 mm rain on the tenth day is summarized in Table 4. Simulated \( h_{S} \)-values in the 20-day period in Table 4 when compared
with published crop tolerance data (Haise and Hagan\textsuperscript{17}), reproduced in Table 5) indicate that the farmer's scheduling rule and the judgement exercised would lead to optimum results with vegetable crops, including the most sensitive varieties.

### Table 6: Critical matrix potentials as reported by Haise and Hagan\textsuperscript{17}

<table>
<thead>
<tr>
<th>Crop</th>
<th>$h_c (m)$ = tolerance in terms of matrix potential</th>
</tr>
</thead>
<tbody>
<tr>
<td>Celery</td>
<td>-2 to -3</td>
</tr>
<tr>
<td>Potatoes</td>
<td>-3 to -5</td>
</tr>
<tr>
<td>Lettuce</td>
<td>-4 to -6</td>
</tr>
<tr>
<td>Carrots</td>
<td>-5.5 to -6.5</td>
</tr>
<tr>
<td>Cabbage</td>
<td>-6 to -7</td>
</tr>
</tbody>
</table>

Use of the program to improve scheduling: If in conventional scheduling, $e_g$ is estimated by applying crop factors to potential evapo-transpiration estimated by the Penman formula from field observations of climate, e.g. as in a farm advice service by Kincaid and Heerman\textsuperscript{18}, the resulting soil moisture profiles can be simulated by the computer program, using the estimated $e_g$ and actual rainfall and irrigation. The success of the scheduling can be assessed by considering the computed $h_c$ and improved by changing scheduling decisions as experience is gained.

Continuous records of estimated evapo-transpiration and computed $h_c$, produced by simulation, will permit meaningful correlation with tensiometer readings at selected monitoring points in the field. Comparative data acquired in this way can be used as a research tool for updating climatological and crop factors. Updating of factors was found to be desirable by authorities on the subject e.g. Nieuwoudt et al\textsuperscript{19}.

Table 6 shows simulated irrigation in the Phillipi field in which a 10-day period with $e_g = 3$ mm was followed by 10 days with $e_g = 7.5$ mm (both periods without rainfall).

In Tables 4 and 6 the computed $h_c$ was acceptable at all times. Had $N_g = 2$ days instead of one day been adopted for the irrigation rule, $h_c$ would not have been so favourable, with revision possibly required to satisfy crop tolerances.

### Prediction of the potential intake rate of loam: A computer run of 24 hours duration was made with a water application rate of 5 mm/h throughout the period. This simulated a field with a 5 mm/h areal mean sprinkler system, at points receiving the average application. The purpose of the simulation was to determine the longest duration of irrigation with the installed system that would not produce surface runoff at the selected points. The appropriate hydro-pneumatic characteristics

### Table 4: Irrigation of sand: Scheduling and simulation with $e_g = 7.5$ mm on all days and 45 mm rain on day No 10

<table>
<thead>
<tr>
<th>T (day No)</th>
<th>I</th>
<th>Simulation (end of day results)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-h_c$</td>
<td>$l_q$</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>2.09</td>
</tr>
<tr>
<td>1</td>
<td>12.5</td>
<td>1.75</td>
</tr>
<tr>
<td>2</td>
<td>12.5</td>
<td>1.65</td>
</tr>
<tr>
<td>3</td>
<td>12.5</td>
<td>1.62</td>
</tr>
<tr>
<td>4</td>
<td>12.5</td>
<td>1.58</td>
</tr>
<tr>
<td>5</td>
<td>45</td>
<td>0.83</td>
</tr>
<tr>
<td>6</td>
<td>10</td>
<td>2.20</td>
</tr>
<tr>
<td>7</td>
<td>12</td>
<td>2.20</td>
</tr>
<tr>
<td>8</td>
<td>12.5</td>
<td>1.77</td>
</tr>
<tr>
<td>9</td>
<td>12.5</td>
<td>1.58</td>
</tr>
</tbody>
</table>

1. The soil moisture in the least favoured part was simulated, leading to results as tabulated under the headings:

   $h_c (m)$ = matrix potential at 250 mm depth

   $l_q (mm)$ = drainage to below the rooting zone

   $d_g (mm)$ = water content in the rooting zone

2. $I (mm)$ = average over area irrigation, as scheduled by Eqn 74, or rainfall on day 10.

3. $I = 0$ when $d_g$ at scheduling = conventional field capacity

4. Figures in brackets where the scheduler presumed that the storage in the rooting zone was greater than normal owing to preceding rain.

During days No 3 to 9 and 13 to 19 (both inclusive) $d_g$ approached the values simulated for days No 9 and 19: asymptotically, with equality in rounded-off values as from days No 5 and 17 respectively.

**Fig 6:** Simulation of an intake rate test in which water is applied by sprinkler equipment to the model loam

of loam were applied in the computer program, with the initial condition equal to a nominal field capacity profile derived as described in the previous problems where sand was irrigated. Fig 6 shows the computed results.

Fig 6 shows that 10 hours water application (or 12 hours in an emergency) would be permissible with the soil characteristics of the model loam and the sprinkler irrigation system.

### Conclusions

1. In all except very unusual situations it is incorrect to apply the diffusivity analogy. Darcy’s Law, with hydraulic gradients derived from relative saturation and matrix potential profiles, is correct in all situations.

2. A computer program for two-phase vertical flow has been successfully developed here. It takes into account hysteresis in matrix potential – with water application by irrigation, rainfall and aquifer

### Table 6: Irrigation of sand. Scheduling and simulation in a situation without rain, with $e_g = 3$ mm on the first and $e_g = 7.5$ mm on the second 10 days

<table>
<thead>
<tr>
<th>T (day No)</th>
<th>I</th>
<th>Simulation (end of day results)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1.47</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>1.52</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>1.58</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>1.60</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>1.68</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>2.92</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>2.92</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>2.92</td>
</tr>
<tr>
<td>8</td>
<td>5</td>
<td>2.92</td>
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<td>9</td>
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<tr>
<td>10</td>
<td>5</td>
<td>2.92</td>
</tr>
<tr>
<td>11</td>
<td>5</td>
<td>2.92</td>
</tr>
<tr>
<td>12</td>
<td>5</td>
<td>2.92</td>
</tr>
<tr>
<td>13</td>
<td>5</td>
<td>2.92</td>
</tr>
<tr>
<td>14</td>
<td>5</td>
<td>2.92</td>
</tr>
</tbody>
</table>

1. Notes 1 to 3 of Table 4 are applicable, mutatis mutandis.

2. The simulations, results of which are listed in Tables 4 and 6, were planned to demonstrate the degree of approximation involved when applying a conventional field capacity curve soil moisture deficit concept by Eqn 74, instead of soil moisture flow theory. In terms of the former all computed results at the end of days with $e_g = 7.5$ mm are presumed to equal those on day No 9 in Table 4 ($d_g = 32.2$ mm). This is not in agreement with the tabulated results based on soil moisture flow theory, the greatest deviations occurring on days No 10, 11 and 12 in Table 4 after rain and on days Nos 10 and 11 in Table 6 after a period with small water applications in cooler weather.

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recharge - and water loss by evapo-transpiration, surface runoff and drainage. All sources of instability in computation were eliminated and perfect water balance in computed results was maintained at all times.

3. It was found that the daily irrigation rule developed and very successfully used by an irrigator of sand and vegetable crops at Philippi, when applied in the computer program together with estimates of the local soil, climatic and crop-related conditions, would lead to the prediction of optimum production results similar to those achieved in practice.

4. Decisions in conventional scheduling can be improved by taking cognizance of results of simulation by the computer program run in parallel with the scheduling.

5. The computer simulation is a research tool by which crop and climatic factors applied in scheduling can be updated.

Acknowledgements

This paper is a report to the engineering profession on the research performed by myself at the University of Stellenbosch for a PhD degree on the subject ‘Two-phase analysis of vertical flow in irrigation and aquifer recharge’. Grateful acknowledgement is made to my supervisor, Prof. L. Hiestra, and to Prof. A. Coetsee and the University of Stellenbosch for support given and facilities made available.

References


Appendix: Basic definitions and notation

1. Capillary fringe: A part of the unsaturated zone separated from the remainder by an interface at which air entry resistance occurs.
2. Crop factor: A constant based on research by which potential evapo-transpiration (qv) is multiplied in scheduling (qv).
3. Hysteresis phenomenon in matrix potential (ψm): In a drying test on a sample of soil, starting with a ψm and ending with a ψm, both as defined below, matrix potential can be monitored and expressed as a function h(ψm), where h denotes the matrix potential and the subscript D drying. If the process is reversed until ψm = ψm is restored, a function h(ψm) can be determined where W denotes wetting. The two functions differ owing to the hysteresis phenomenon. Curves plotted in terms of the functions have the same end points and together form a hysteresis loop.
4. Matrix potential: h, (m) = the contribution made by capillary forces in soil moisture to its energy, expressed as a head of water. It is negative in the unsaturated zone and zero at relative saturation s = s_c.
5. Potential evapo-transpiration: The consumptive use of soil moisture in a field with a standard crop at a specified stage of maturity when adequate soil moisture is maintained (applied in the Penman formula).
6. Relative saturation s: s = the fraction of void space in soil occupied by water.
7. Saturated soil: the critical saturation or the upper limit when in a wetting process a remaining fraction of air cannot be displaced by water. s_p = the residual saturation when, in a drying process, the water is considered to become immobile, regardless of a prevailing hydraulic gradient due to gravity or matrix potential.
8. s_f = the relative saturation at the interface between a capillary fringe and less saturated soil.
9. Scheduling: The calculations made to determine the depth of irrigation water to be applied at prescribed scheduling time.
10. Soil: According to the word usage of ‘soil science’, soil refers to sand. The terms ‘porous medium’ and ‘aquifer material’ are used where appropriate.
11. Two-phase flow: In the unsaturated zone two fluids, water and air, jointly occupy the voids in the soil, separated by surfaces of the wetting fluid (water) held in shape and position by surface tension.
12. Water balance: In a conceptual column of soil with the flow of water confined to the vertical direction, the increase of water storage in the column equals the volume of water added minus the volume extracted and drained, all in the same time interval. Maintenance of water balance is the compliance with this requirement in simulation.
13. etc.