Pressuremeter interpretation by progressive relaxation

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Introduction
A need exists in geotechnical engineering for a model of soil stress-strain behaviour that is simple in concept and that has easily obtainable parameters. Because of this need, the linear isotropic elastic model, with just two constants, is widely used in practice. A significant improvement in performance over that model, while remaining simple in concept, is offered by one that allows for variable moduli.

This paper presents an easily applied method of obtaining the parameters for a model in which the bulk and shear moduli of the soil depend upon the current stress-strain states.

Method
The soil around the pressuremeter is assumed to be divided into annular rings of uniform thickness, the interfaces between which are numbered as shown in Fig 1. (The thickness and number of these rings are discussed later.) Initially, only one ring of soil is considered to exist round the pressuremeter and the first stage of radial stress $\sigma_r$ is applied to the inside of the ring at radius $r_1$. The movement at the outside of the soil ring is prescribed to be zero and, using Lamé's theory, the radial expansion of the pressuremeter ($\delta r_1$ at radius 1) is calculated. The choice of the elasticity parameters used is described below under 'Material model'.

The calculation of $\delta r_1$ at $r_1$, using Lamé's theory, is carried out in two stages. First the induced radial stress at $r_2$ (that is $\sigma_r$) is found for the applied radial stress at $r_1$ and with the condition that $\delta r_2 = 0$. Second, for the inner and outer radial stresses, $\sigma_r$, and $\sigma_u$, the radial movement at the pressuremeter, $\delta r_1$, is calculated. (Note: the algebraic derivations of these relationships are not complex, but are too voluminous to be included in this technical note. If interested readers contact the author, a copy will be provided.)

A second annular ring is then added and calculations as previously described are applied to both rings, starting with the outside one, in the following manner (use Fig 1 as a guide when reading the next paragraph).

The radial stress at $r_2$ (calculated when $\delta r_1$ was assumed to be zero) is now the applied radial stress on the ring between $r_2$ and $r_3$, and the induced radial stress at $r_2$ is calculated for $\delta r_2$ prescribed to be 0. For $\sigma_r$, and $\sigma_u$, $\delta r_2$ is then found. This value of $\delta r_2$ is now the prescribed movement at $r_2$, used to calculate a new value of $\sigma_r$, induced by $\sigma_r$, the increment of stress applied by the pressuremeter.

Finally, $\delta r_n$, is calculated using $\sigma_r$, and the most recently calculated value of $\sigma_r$.

In the procedure described for two rings, it can be seen that $\delta r_n$ is now free to move, whereas with just one ring, $r_2$ was at the outside of the soil annulus around the pressuremeter and therefore $\delta r_n$ was prescribed to be zero. The relaxation of $\delta r_n$ reduces $\sigma_r$ and hence allows $\delta r_n$ to increase. This progressive relaxation of the body of soil round the pressuremeter continues with the inclusion of further annual rings at the outside. Fig 2 shows the effect on the calculated expansion of the pressuremeter of increasing the number of annular rings of thickness (pressuremeter diameter/8).

The addition of further rings of soil can be stopped when the effect on $\delta r_n$ becomes negligible. In the case of the example shown in Fig 2, this was at a radius of 2.5 times the pressuremeter diameter. Hence the number of annular rings of soil is chosen automatically. Because each ring has its own values of elastic moduli, the thickness of the rings will influence the calculated value of $\delta r_n$. However, this influence is not great and a thickness of pressuremeter diameter/8 is a good compromise between accuracy and speed of calculations.

Material model
The material model considers separately the behaviour of the soil under the hydrostatic and shear components of the stress state acting on it.

Fig 1: Division of soil around pressuremeter into annular rings

Fig 2: Effect of the number of rings of soil around the pressuremeter on its calculated radial expansion under one increment of stress

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Soil behaviour under hydrostatic stress

For the expansion of a cylinder in an isotropic, linear elastic material, there is no net change in the mean direct stress (ie the hydrostatic component of the stress state). However, because the soil behaviour is not linear, there will be a net increase in compressive stress on the soil around the pressuremeter. It is assumed that the behaviour under hydrostatic stress is that given by the results of a laboratory test on an undisturbed sample of the soil. Domaschuk and Wade\(^*\) suggested that the following function can be used to represent that behaviour:

\[
\sigma_m = \alpha \varepsilon_v^\beta
\]

where

\[
\sigma_m = \text{hydrostatic stress} = (\sigma_1 + \sigma_2 + \sigma_3)/3 \\
\varepsilon_v = \text{volumetric strain} \\
\alpha \text{ and } \beta = \text{constants found by plotting the test results on axes of log } \sigma_m \text{ against log } \varepsilon_v
\]

A typical test result and the Eqn 1 functions to represent it are shown in Fig 3.

In the calculation of radial stresses and movements that pertain at the end of a stage of load application by the pressuremeter, the secant bulk modulus, \(K_{sec}\), is needed. An expression for \(K_{sec}\) is derived as follows:

\[
K_{sec} = \frac{\sigma_m}{\varepsilon_v}
\]

But from Eqn 1, \(\varepsilon_v = \left(\frac{\sigma_m}{\alpha}\right)^{1/\beta}\)

Substituting for \(\varepsilon_v\), gives \(K_{sec} = \alpha^{1/\beta} \sigma_m^{(1-1/\beta)}\)

Soil behaviour under shear stress

It has been found by Kondner and Zelasko\(^2\) and Duncan and Chang\(^3\) that a hyperbolic relationship successfully models shear behaviour. This relationship is of the form:

\[
\tau = \frac{\gamma}{G_{int} + \tau_{asympt}}
\]

where

\(\tau\) and \(\gamma\) = the corresponding shear stress and strain on any plane

\(G_{int}\) = initial shear modulus at very low strains

\(\tau_{asympt}\) = asymptotic stress limit of shear stress \(\tau\) that the hyperbolic Eqn 3 approaches.

These parameters are illustrated in Fig 4. The value of the secant shear modulus is then given by:

\[
G_{sec} = \frac{\tau}{\gamma} = \frac{1}{G_{int} + \tau_{asympt}}
\]

It is convenient, for simplicity of expressions, to use the plane on which maximum shear stress and strain occur (assumed coincident) to calculate the value of \(\gamma\) for use in Eqn 4.

The two parameters \(G_{int}\) which is a measure of shear stiffness, and \(\tau_{asympt}\) which is a measure of strength, are required to be found. Both could be found from the pressuremeter test, but because different drainage conditions will exist in the actual design situation, where movements need to be predicted, the shear strength parameters used then in the soil model will need to be appropriate for that situation. For this reason it is proposed that \(c\) and \(\phi\) from laboratory shear tests with drainage similar to the pressuremeter test be used to interpret it. The same type of laboratory test would then be used to give \(c\) and \(\phi\) for the drainage conditions of the design situation.

The relationship between \(\tau_{asympt}\) (ie maximum shear stress on Mohr’s circle) and \(c\), \(\phi\) is illustrated in Fig 4 and is given by:

\[
\tau_{asympt} = c \cos \phi + \frac{(\sigma_1 + \sigma_2)}{2} \sin \phi
\]

This leaves \(G_{int}\) as the only unknown parameter from the soil model to be found from interpretation of the pressuremeter test. Its value is found by varying it in a process of trial and error, until the pressuremeter results graph of radial stress against radial expansion has been reduced by the \(c_{v1}\) against \(\delta_{v1}\) graph from the calculations described above.

Example of application

Fig 5 is the graph of probe pressure (corrected for membrane stiffness) against radial expansion of a ‘Camkometer’ self-boring pres-
suremeter in a stiff clay (the Gault clay in eastern England). To interpret it, it is first necessary to choose the appropriate degree of consolidation curve from the isotropic consolidation test results shown in Fig 3, which are from a test done on a sample from the same site and depth. The assessment of the degree of consolidation taking place during a pressuremeter test is very difficult. A simplified approach has been adopted as follows:

1. Assume that an instantaneous rise in radial stress at the pressuremeter of 100 units is carried entirely by a rise in pore pressure and the radial distribution of the pore pressure is as given by linear elasticity theory (i.e. Lamé's theory).
2. Using the finite difference solution for radial flow consolidation and assuming free drainage boundaries at the pressuremeter and at four diameters away from the pressuremeter, calculate the dissipation in pore pressure at radial intervals between those boundaries over the period of time that the test was done.
3. Because more of the measured radial movement of the pressuremeter is caused by the consolidation of the soil close to it than of that further away, a weighted average is found for the degrees of consolidation calculated in stage 2. The method of weighting is according to the elastic distribution of radial stress.

From such a weighted assessment, the appropriate degree of consolidation for the test in this example is 20 per cent, hence the 20 per cent points on Fig 3 were used to get $c$ and $\beta$ for the bulk modulus function, Eqn 2. Next it is necessary to choose values of shear strength parameters $c$ and $\phi$ that are appropriate to a 20 per cent dissipation of the change in pore pressure that is induced in a triaxial shear test. Again this is very difficult, but from available test information on the Gault clay, values of $c = 130$ kN/m$^2$ and $\phi = 0$ were estimated.

The calculated simulation of the pressuremeter test is shown on Fig 5 for three values of $G_{\text{ult}}$. In a practical situation where soil movements are to be predicted, the majority of the soil will be well below failure, say in a range of stress up to about 350 kN/m$^2$ on Fig 5. For this range, $G_{\text{ult}} = 60$ MN/m$^2$ gives as good a fit as would be needed in practice. The soil model would then be used (probably with the finite element method) with this value of $G_{\text{ult}}$ and the values of $c$, $\beta$, $c$ and $\phi$ appropriate to the drainage conditions that will exist in the design situation where movements are to be predicted.

For the five increments of stress shown in Fig 5, a calculation of the corresponding radial expansions took just two minutes with compiled basic on an IBM PC. With this degree of speed and operational convenience, it is quite easy to examine the sensitivity of the pressuremeter test to variations in the other parameters in the soil model as well as $G_{\text{ult}}$.

Conclusions

1. The progressive relaxation method of pressuremeter interpretation can be applied with a variable moduli model of soil behaviour.
2. By the inclusion of laboratory test results in the soil model and by trial and error choice of the initial shear stiffness modulus, the pressuremeter test result can be closely reproduced.
3. The sensitivity of the reproduction of the pressuremeter test result to variation in the parameters that define the soil's behaviour can be calculated easily and quickly.
4. Having found the parameters for the soil model that enable the pressuremeter test to be reproduced, parameters for the drainage conditions that apply to the design situation can then be substituted in the model.

References


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These days the old van Stadens River Bridge seems a little modest compared to the spectacular concrete arch above it. But make no mistake, it's as stout today as it was over fifty years ago.

Solidly built with PPC cement, it's stood up to virtually anything man or nature could dish out. Including the floods of 1968.

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