Links between content knowledge and practice in a Mathematics Teacher Education course: A case study

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Abstract
This qualitative study examined the link between content knowledge and classroom practice from the perceptions of two university lecturers. The study was contextualised at a higher education institution in South Africa where the two university lecturers were lecturing to a second year undergraduate teacher trainee class (n = 78). The research was conceptualised in terms of Vygotsky’s educational theory and the process of scaffolding. Questionnaires in which these lecturers expressed their views on content knowledge in general were administered to them. Video recorded lessons on rates of change in Calculus were observed to triangulate actual lesson instruction and their views on content knowledge and classroom practice. Data yielded by these research instruments confirmed certain assumptions and literature claims. The study revealed that the two university lecturers portrayed a strong link between a lecturers’ content knowledge and his/her classroom practice.

INTRODUCTION
Since the publication of the Presidential Education Initiative (PEI) Report (Taylor and Vinjevold as cited in Brodie 2004, 65), ‘the issue of mathematics teacher knowledge has been on the research and teacher education agenda in South Africa’ (Brodie 2004, 65). According to the PEI report it was found that teachers needed a better conceptual knowledge of mathematics in order to assist learners to gain access to the mathematical content. Also in America, according to Wu (2005, 1), there has been a trend in mathematics education, which she found quite alarming, to avoid the importance of content knowledge. Wu believed that ‘sound pedagogical decisions are based on sound content knowledge’ (Wu 2005, 1). This study therefore embarked to shed some light on the relationship between content knowledge and practice in the context of teaching at a higher education institution.
This study was part of a National Research Foundation (NRF) project that tested the implementation of activities designed by teacher education units at three South African universities. This research focused on the link between content knowledge and practice in specific relation to the teaching of calculus. Data from the ‘calculus project’, in the form of videotaped lessons of two university lecturers teaching calculus to an undergraduate teacher trainee class was analysed. Other data such as the interviews conducted with the university lecturers was retrieved during the validation phases of the study.

Current research needs to focus on providing an analysis of teaching practices and the mathematical knowledge that is required to improve and also to sustain these practices (Ball et al. as cited in Brodie 2004, 66). Ball et al. also argued that there should be a strong link between the mathematics that teachers have to learn and the activities that define their practice in the classroom. In contrast to this, the PEI report argued that too much emphasis had been placed on teaching methodology in South African curriculum initiatives, at the expense of content knowledge that needed to be taught and learned.

Adler (2005) focused on the complex issues of the teaching and learning of mathematics. She felt it was important that we understood ‘how to make mathematics learnable by all children’ (Adler 2005, 2). Her area of interest is to know more about the support and mathematical preparation that teachers receive in order to make them more efficient and skilful in the classroom context.

Even (1990) also subscribed to this notion and argued that the way mathematics is taught is important for mathematics educators currently. The emphasis in recent times is to teach in such a manner that learners understand and so that meaningful learning takes place. The role of the teacher is to assist the learner to understand the subject matter. ‘But in order to do so the teachers themselves need to have a solid knowledge of the subject matter’ (Even 1990, 521). A teacher that has a solid knowledge of the content that he is teaching will be able to impart this in a more meaningful manner.

Ensor looked at the possible explanation for the disjuncture between ‘what individual teachers said about how they thought children learn, and the classroom practices of those same teachers’ (1999, 2). Her explanation emerged from a two year longitudinal study where she researched the relationship between teacher education and classroom practice. Her findings were inconclusive and so it was felt that further research on this is required.

Margolinhas et al. (2005) used case studies in order to deepen their understanding of what teachers learn from their classroom experience. They focused on the teachers’ didactic knowledge in relation to the observation in the classroom. In all these studies research was carried out in schools. This study is different in that it is carried out in higher education.


D. Brijlall and V. Isaac

AIM AND CRITICAL QUESTIONS

The aim of this research was to generate thick interpretations of data in the form of videotaped lessons, and interviews, in order to explain the lecturers’ perceptions of the link between content knowledge and the activities that they engaged with in the classroom. To unpack this aim the following critical questions were explored:

(a) What were the lecturers’ perceptions of content knowledge?
(b) Did the lecturers engage their students in dialogue around the activities that occurred in the class and how did the nature of the dialogue influence the lecturer’s classroom practice? and
(c) How did the lecturers view the link between content knowledge and classroom practice?

SHULMAN’S PRESIDENTIAL ADDRESS OF 1986

Shulman’s inquiry into teacher education entailed him looking at literature that dated back to 1875, where he looked at tests that teachers were given. These tests demonstrated how teacher knowledge was viewed and defined. There were certain assumptions that underlined those tests and those were:

the person who presumes to teach subject matter to children must demonstrate knowledge of that subject matter as a prerequisite to teaching. Although knowledge of the theories and methods of teaching is important, it plays a decidedly secondary role in the qualifications of a teacher (Shulman 1986, 5).

This emphasis on content knowledge contrasted with the emerging policies of the 1980’s where the emphasis was on pedagogic knowledge, the knowledge to be able to teach.

THE PRESIDENTIAL EDUCATION INITIATIVE (PEI) REPORT

Education systems presently not only act as vehicles for transformation and change but must also produce citizens that ‘will enable their countries to become globally competitive’ (Taylor and Vinjevold 1999, 2). This challenge was faced by South Africa’s first democratic government. In order to meet this challenge and also to make provision for an education system that is equitable, of high quality and also one that is relevant to all its citizens, the PEI was born. The PEI report was responsible to a large degree, in South Africa, for initiating and catapulting research on content knowledge. In this regard research was conducted by, amongst others, Adler, Slominsky, Reed, Ensor, Long and Brodie.
DEFINITION OF KEY CONCEPTS

This study applied the following terminology within the constraints of the definitions provided:

(a) content / subject matter knowledge (C/SMK)

These two concepts are used interchangeably in this research and mean the same thing. Content knowledge refers to the knowing about a subject, the disciplinary knowledge of a subject. ‘Mathematical content knowledge includes information such as mathematics concepts, rules and associated procedures for problem solving’ (Chinnapen 2003, 1).

(b) pedagogical knowledge (PK)

Pedagogical knowledge refers to the broad knowledge that a teacher requires in order to be effective in the classroom. This includes content knowledge, knowledge about how to teach, knowledge about pupils and how they learn, knowledge about the curriculum and knowledge about discipline and classroom management.

(c) conceptual knowledge

This term is used by Adler, Slominsky and Reed (2002) and refers to the special way that a teacher uses the mathematical content in order to teach mathematics. Adler et al. draw a distinction between the way a mathematician would view the mathematical content and the way a mathematics teacher views mathematical content. The teacher has to impart content knowledge to his students.

(d) pedagogical content knowledge (PCK)

This term was first used by Shulman. It refers to a blend of content knowledge and pedagogical knowledge. This includes understanding why some children experience difficulties learning a concept whilst others find it easy to understand. PCK includes appropriate teaching approaches and questions the quality of explanations that teachers give during a lesson. This study does not draw a distinction between a teacher, an educator or a lecturer. These terms are used to refer to those that impart knowledge (MKO) to their pupils, learners or students. The terms learners and pupils are used to refer to children in a school and students refer to those studying at a university.

(e) common content knowledge (CCK)

This is the mathematical knowledge and skill used in settings other than teaching (Ball et al. 2008).

(f) specialized content knowledge (SCK)

This domain of knowledge is the mathematical knowledge and skill unique to teaching (Ball et al. 2008).
Psychology refers to perception as the act of interpreting a stimulus by one or more of our senses (Sperling 1967, 36). The stimulus that I presented to the lecturers was the interview questions. The mechanics for receiving stimuli are similar for different individuals. However different individuals may interpret the stimuli in different ways.

**THEORETICAL FRAMEWORK**

This study was conceptualized in terms of Vygotsky’s educational theory and also the process of scaffolding. According to Vygotsky’s theory ‘learning leads to the development of higher order thinking’ (Dahms et al. 2007, 1). Vygotsky believes that learning occurs through language and social interaction. The zone of proximal development (ZPD) is central to Vygotsky’s view on how learning occurs. He describes this zone as the distance between the actual developmental level of a learner which is determined by independent problem solving and the potential developmental level of a learner which is determined by problem solving under the supervision of a teacher or a more capable peer.

‘Vygotsky defined those who teach as the ‘More Knowledgeable Other’ (MKO)’ (Dahms et al. 2007, 2). The MKO refers to anyone who has a higher level of understanding than the learner. The key characteristic of the MKO is that their knowledge of the topic that is being taught must be greater than that of the learner. In this way they can raise the competence of their learners. The MKO shares his knowledge with his students in order to bridge the gap between what they know and what they do not know. Once the student expands his knowledge the ZPD shifts. The ZPD is always changing as the students gain more knowledge. In my study the MKO would be the two university lecturers and it would be interesting to note their perception of how content knowledge was linked to the competencies of their students in order to allow a shift in the ZPD of their students.

According to Vygotsky (as cited in Dahms et al. 2007, 2) the ideal role of the teacher is to provide scaffolding to assist the learner with their tasks. ‘The term “scaffolding” was developed as a metaphor to describe the type of assistance offered by a teacher or a peer to support learning’ (Lipscombe et al. 2008, 3). Scaffolding is a process whereby the teacher helped a student to grasp a concept or master a task that he was initially unable to do. Assistance was only provided to those skills that were beyond the student’s capability. The student may make errors, but with feedback and prompting, the student is able to reach the required response. Once the student can perform the task on his own the teacher then removes the scaffolding which allows the student to work independently.

Scaffolding is actually a bridge used to build upon what students already know to arrive at something they do not know. If scaffolding is properly administrated, it will act as an enabler, not as a disabler. (Benson as cited in Lipscombe et al. 2008, 3).
PRINCIPLE OF SCAFFOLDING

According to Zhao and Orey (as cited in Lipscombe et al. 2008, 5) scaffolded instruction has six general elements namely: ‘sharing a specific goal, whole task approach, immediate availability of help, intention assisting, optimal level of help and conveying an expert model’ (Lipscombe et al. 2008, 5)

(a) Sharing a specific goal – It is the responsibility of the MKO to establish a goal and share it with his students. Allowing for input from the students on the shared goal enhances their intrinsic motivation. It also assists the learner to master the goal that has been set, by developing new skills. In this way, learning is also enhanced and the student sees that he can attain success by putting in the required effort.

(b) Whole task approach – The focus of this approach is on the ultimate goal that is to be achieved. In this approach each task is seen as how it relates to the ultimate goal. This approach is only effective if the student does not experience any difficulty in any of the minor tasks in order to achieve the ultimate end result.

(c) Immediate availability of help – In scaffolded instruction success is important in order to control the frustration levels of the students. If the MKO provides assistance and support timeously, then the student experiences success and this motivates the student to be more productive.

(d) Intention assisting – In the scaffolding process it is important to provide assistance to the learner’s present focus in order to assist him with his current problems. In this way a more productive learning environment is developed as the learner pursues his current task. Sometimes, however, it is necessary to redirect the learners thought processes, if they do not coincide with the current task at hand. The MKO needs to be aware of the different methods of completing a task and if the learner’s method allows him to complete the task in a different way then the MKO must accept this method and allow the learner to proceed with the least amount of assistance. If however the learner needs constant assistance then the MKO could turn to coaching to assist the learner.

(e) Optimal level of help – The level of assistance that is provided to the learner must match what he is able to do. The learner must get just enough assistance in order to overcome the current obstacle, but the level of assistance should not hinder the learner from contributing and participating in the learning process of that particular task (Lipscombe et al. 2008, 7).

(f) Conveying an expert model – An expert model provides the learner with an example of how to accomplish a particular task. The techniques that are to be used for completing the task are clearly expressed.
DATA GENERATING INSTRUMENTS

Primary data was generated by conducting semi-structured interviews with the two university lecturers and secondary data was obtained by observing the videotaped lessons. Later another set of interviews was required to clarify and fill in gaps identified. The activity sheets were based on what research in calculus findings suggested texts should satisfy and Vygotsky’s educational theory involving the process of scaffolding.

DISCUSSIONS AND FINDINGS

Analysis of observation

The lecturers were observed lecturing to a class of second year undergraduate students. There were seventy eight students in the class of which fifty three were male and twenty five were female. The course dealt with Differential Calculus. The topic under discussion was ‘rates of change’. An extract from the activity sheet is presented in Figure 1. This is to provide a glimpse of the tasks students confronted in the activity sheets. Two video recordings of lessons conducted by lecturer 1 and lecturer 2 were observed. The first lesson started with the students working on the solutions to the questions that were given to them in the activity sheet. The students were given time in class to work on the solutions.

Consider a linear function \( f(x) \) and a curve \( g(x) \).

1. a) If point \( C(6, 8) \) lies on the graph of \( f(x) \), is the gradient of \( \frac{4}{3} \) the line ?
   b) Can you use point A only to find the gradient of the line \( y = f(x) \)? Explain.
   c) Use points A and B to find the gradient of the linear function, and show your working.
   d) Move point B to the right of A and then to the left of A, so that the gradient of the line \( AB \) is still the same as it is now.
   e) If we keep point A as it is, explain how you could move point B so that
      i) the line \( AB \) is steeper than it is now
      ii) the line \( AB \) is less steep than it is now
      iii) the line \( AB \) has a negative gradient
      iv) the y-value of the linear function increases three times as fast as the x-value
      v) the y-value of the linear function decreases twice as fast as the x-value
   f) Use the gradient of the line to find \( p \) if the point \( (12, p) \) lies on the graph of \( f(x) \).

Figure 1: An extract of the activity sheet
The students were seated in a lecture room. The desks were all single desks that were arranged in rows facing the front of the lecture room. There were three rows of desks, the row on the left had three desks grouped together, the middle row had two desks grouped together and the row on the right had two desks grouped together.

Frame 1: The groups that students formed.

The students were either working on their own or in groups. The groups the students formed on their own by either turning around and working with students behind them or by working with students that were sitting next to them (as illustrated in Frame 1).

As the students worked on their responses both lecturer 1 and lecturer 2 walked around the lecture room to either assist the students or check on what they were doing. Several times both lecturer 1 and lecturer 2 stopped at the desks of students and assisted them with their queries. This can be observed in Frame 2.

Frame 2: Lecturer assisting students.
In this way students were provided with immediate assistance, this in keeping with Zhao and Orey’s (as cited in Lipscombe et al. 2008, 5) scaffolded instruction. Students also raised their hands to get the attention of either lecturer 1 or lecturer 2 to answer their queries or questions. After about twenty minutes lecturer 1 asked for a volunteer to come to the front of the class and work out question one on the board. The students were initially reluctant but after some persuasion and coaxing a student eventually came to the front of the class and attempted this question on the board. Lecturer 1 guided him as he was working out the solution. Thereafter another student worked out question 2 on the board. Lesson two also began in a similar manner, but this time the students were working on the solution to a follow-up question. Only lecturer 2 was present at this lecture. Here again students volunteered to work out the subsections of question 3 on the board. Lecturer 2 encouraged the students to come to the front of the class and also assisted and guided them when they were working out the solutions on the board (see Frame 3). Both the lecturers socially interacted with the students to promote learning. Using language and social interaction the lecturers engaged with their students in order to promote learning. This is in keeping with Vygotsky’s learning theory. Vygotsky’s learning theory advocates that learning is enhanced through the social interaction between the student and a teacher. Vygotsky views the teacher as the MKO who is able to lift the student’s achievement level.

Frame 3: Student working on board, assisted by lecturer.

The lecturers also lifted the performance of the students that they were supervising by providing immediate assistance, this in keeping with the scaffolding approach. The ‘immediate availability of help’ is one of the six general elements of the scaffolding approach.
Participant profiles

Profile of lecturer 1

Lecturer 1 is a mathematics lecturer at a tertiary institution in South Africa. He has been teaching for twenty five years. His teaching experience includes teaching at a high school, a teacher training institution and a university. His academic qualifications include a Bachelor of Science degree, a Bachelor of Science Honours degree, a Bachelor of Education Honours degree, a Masters of Science degree and a Doctorate in Pure Mathematics. Lecturer 1 is thus an experienced teacher who is highly qualified.

Lecturer 1 enjoys teaching, he enjoys teaching the mathematical content and he also enjoys putting ideas across to others and seeing their face light up when they actually understand what I’m getting across to them.

This statement reflected a passion for teaching and also a commitment to imparting knowledge to his students.

Lecturer 1 also engaged in dialogue with his colleagues so as to improve and adjust his teaching to suit the calibre of students that he is exposed to and also to gain insight and knowledge about his students.

But even on the informal level when we chat in corridors and at meetings we actually speak about students and what transpires in our lecturers and that does have an impact on our teaching, in the sense that we get to know our students more. If the student has been dealt with by lecturers in the past, so we get some past history of the students. At the same time it also helps us to actually check our teaching in the sense that we know the calibre of the students that we are dealing with so we know how to adjust our teaching in that respect.

Although lecturer 1 had not received any formal training on developing successful relationships with his students, he had actually developed this in an informal manner from his interactions with students over the years. This is illustrated by what he said:

The actual relationships that I encountered with students transpired during my years of teaching, that was developed incidentally

From the video recordings it was observed that the students reacted in a positive manner to lecturer 1. If they had any queries or problems they were comfortable to ask him for assistance. The students also seemed quite relaxed in the classroom and were allowed to work at their own pace without much pressure. This also displayed the rapport that lecturer 1 shared with his students.
Profile of Lecturer 2

Lecturer 2 is also a mathematics lecturer at a South African institution. She is an experienced lecturer who has been teaching for fifteen years. Her qualifications include a Masters degree in Mathematics and a Doctorate degree in Mathematics Education.

She enjoyed interacting with students and also believed that her teaching allowed her to learn more, ‘because as you teach you learn more’.

This enjoyment of pupil interaction was also quite evident in the observation of lecturer 2 in the classroom context. She constantly interacted with her students by either asking questions or assisting them in their work.

She also believed that her interaction with her colleagues allowed her to broaden her academic horizons.

I enjoy working with them because of the opportunities to learn, because different people have different perspectives and different strengths. And sometimes you can even look at one problem differently and you learn from the other person’s approach, you learn a different method.

Lecturer 2 is obviously very enthusiastic about learning and developing her skills. Also what was apparent from the above statement is her openness to new and different ideas. She is prepared to learn from other people. Also what came across very strongly in the interview was her passion and commitment to teaching. Apart from her actual responses to the questions, her body language and tone of voice portrayed her passion for teaching.

Although lecturer 2 had not received any formal training on developing successful relationships with her students she believed that successful relationships depended on your commitment to the teaching profession.

In the academic field as well developing successful relationships depends on your own need, your sense of fulfillment, whether you get a sense of fulfillment from developing relationships with students or whether you are more comfortable doing research, or whether you are more comfortable working somewhere else. It’s about your commitment to your job.

Lecturer 2 seemed to be disappointed with some of her colleagues lack of commitment to their profession.

... you see it all the time, you can have ten people in a department and you can see how different people interact at different levels, different people are prepared to go to different lengths to help students and others are not prepared to go even if you send them for training they’ll do the bare minimum.

This statement also demonstrated the type of personality that lecturer 2 has, she is quite emotional and involved in her profession. She is not apathetic but is rather
concerned about the education profession as it currently. This was also evident in
the following comment that she made:

... it’s the same thing, like with teaching, it’s a passion. Are you passionate about
teaching? It’s the problem with our education system as you know because we have
too many teachers who are not passionate about teaching.

PARTICIPANTS’ PERCEPTIONS TO CONTENT KNOWLEDGE

Lecturer 1 defined content knowledge as:

To me content knowledge is knowledge that is pertinent to a particular topic you are
teaching, in other words it does not entail didactic aspects of knowledge. I do not
integrate it with pedagogics. It is dealing with a particular topic and the mathematics
around it.

Lecturer 2 also expressed similar sentiments:

Content knowledge...knowing the mathematics. The content is about the content,
how well you know the content, how it fits in with other topics...about the concepts,
having an understanding of how it works, when it works, knowing interrelationships
within the content.

What came across very strongly was that content knowledge referred to the
actual mathematics that you are required to teach. These sentiments are echoed by
Kilpatrick et al. who believe that content ‘includes knowledge of mathematical facts,
concepts, procedures, and the relationships among them; knowledge of the ways
that mathematical ideas can be represented; and knowledge of mathematics as a
discipline’ (2001, 371). These lecturers responses also agree to Chinnapen’s (2003,
1) view to content knowledge when he stated that content knowledge refers to the
knowing about a subject, the disciplinary knowledge of a subject.

When content knowledge is looked at in quantitative terms then both the lecturers
are adequately qualified to teach the course. When asked: ‘What educational courses
or training have you taken or received to teach Calculus?’ Lecturer 1 explained:

For this topic is part of the first year, university undergraduate at the B.Sc. level. You
do calculus, a whole course in calculus, and then later on you actually study advanced
topics in calculus at a higher level like in Measure Theory and Real Analysis.

Lecturer 2 on the other hand refers to the methodology of teaching and comments:

I’m sure we must have done methods in teaching maths at university.
Lecturer 1 also believed that his content knowledge was adequate to teach the particular course since his content knowledge of calculus far exceeded that which he had to teach since Measure Theory is a study of various types of integrals other than those dealing with Riemann Sums. This idea is supported by Long (2003) and Hilton (as cited in Long 2003) who argued that it was beneficial and advantageous for the teacher to know content that extended beyond the curriculum in order to answer pupils queries. Lecturer 1 had studied calculus up to the honours level and this was reflected by the following comment:

I would say yes because, as I mentioned, the depth of the calculus course. Calculus is not just plain first year differential and integral calculus one sees in textbooks at the classical course given to first year students. In fact calculus has been studied much deeper. As I mentioned if you look at it from an analytical point of view in where you do real analysis in second year and third year courses, where you really have to look at calculus at an advanced level.

This response alludes to the depth of subject matter knowledge that lecturer 1 has on this particular topic. This depth of knowledge contributes to the high level of conceptual thinking experienced by students. The need for in-depth content knowledge ‘for teaching is of primary importance, for without this, teachers would not be able to engage their learners in high-level conceptual thinking’ (Adler et al. 2002, 136).

Lecturer 2 on the other hand did not specialize in calculus, but nevertheless she also believed that her content knowledge was adequate to teach the course.

That class that I was teaching calculus I just went in for a few weeks as part of the project. I didn’t have any hesitation in managing because I knew I would know the calculus, except that I haven’t taught it for a while. No I don’t have any problems in maths with content knowledge.

Also from the observation of their lessons it was quite evident that both lecturer 1 and lecturer 2 were quite comfortable to teach calculus to their classes. They could answer student’s queries and questions adequately. They were also able to guide students’ thinking to bring them to the correct answer. To demonstrate this reference is made to one of the tasks in the activity sheet. The exchange between lecturer 1 and a student exemplifies this point:

Lecturer 1: “Oh, right, so what is your answer then to this question?” (referring to question 1). “Would you say that the statement is always true, never true or sometimes true? So you say it’s always true. So what do you mean by always true? What is always true?”

Student: “We said it’s always true according to this function.”

Lecturer 1: “Very good. He says that according to this function it’s always true.”
Student: “Ya.”
Lecturer 1: “So, can there be another graph? There can be another graph.”
Student: “Yes.”
Lecturer 1: “For which this will be true?”
Student: “No it’s not true.”
Lecturer 1: “So it might not be true. So what will be our choice among our three options?”

In this way the student arrived at the correct answer. This exchange is also a good example of the scaffolding process. There was an immediate availability of help to the student from lecturer 1. Lecturer 1 also redirected the student’s thought processes to bring him to the correct solution.

In order to keep his knowledge of calculus current and updated Lecturer 1 attends conferences, presents research papers, analyses students’ work and also reads current literature on the topic.

“If I was printing a paper I just read by another academic” (points to a research paper on his desk), “and I will read this paper obviously and see how this topic is being taught internationally and what successes they have gained so I can implement similar strategies in my teaching.”

Lecturer 2 on the other hand had to prepare for the calculus classes as she had not taught this topic recently.

When I was asked to teach that course I looked at three textbooks, I went over them properly. I looked at Dr. X’s notes, I worked out every possible question before I taught.

PARTICIPANTS’ CLASSROOM PRACTICE
‘The type of classroom climate generally considered to best facilitate pupil learning is one that is described as being purposeful, task-orientated, relaxed, warm, supportive and has a sense of order’ (Kyriacou 1991, 65). This was evident from my observation of the lessons and also the activity sheets that the students were provided with. There was an atmosphere of purpose that pervaded the lecture room. The activity sheets provided the questions that the students were purposefully engaged with. The students were actively working on the solutions to the problems that they were given. The activity sheets also orientated the students to the tasks at hand. The students were also relaxed as they worked in groups that they had formed on their own, yet order prevailed. Both lecturer 1 and lecturer 2 have warm personalities and
interacted with the students in a congenial manner. Students were encouraged and assisted where necessary.

Lecturer 1 generally introduced a new topic with a problem or mathematical task.

At the commencement of the lecture I always present them maybe a small mathematical task that they have not done before but they have some idea about how to get along, but they probably would not solve it. But the whole intention would be to say at the end of the lecture is that they can now solve the problem. I don’t know if I’m clear. Ya, a simple example I could give you, for example in the grade eleven class, the child can solve the trinomial using the factor method. So after you do the factor method you will probably throw one that does not factorize, so that at the end of the lesson he will learn how to use the quadratic formula, so at the end of the lesson he has now learned a new technique.

From the video recording, it was observed that lecturer 1 demonstrated this. One group of students, when working with question 1 indicated that they had failed to solve the problem. Lecturer 1 then asked them to explain geometrically what the derivative meant. After a discussion with the students he then asked them to evaluate \( f(-3) \) and told them that they should now be able to apply themselves and solve the task.

This relates to Shulman’s ideas on classroom practice as he stated that ‘teaching necessarily begins with a teacher’s understanding of what is to be learned and how it is to be taught’ (Shulman 1987, 7). Shulman believes that teachers know something that is not understood by others and that they can transform their understanding into pedagogical representations and actions. These are ways of talking, showing, enacting, or otherwise representing ideas so that the unknowing can come to know, those without understanding can comprehend and discern, and the unskilled can become adept (Ibid., 7).

Lecturer 2 would either use a problem to start a new topic or the students would be expected to read their notes, which are given to them before the lesson, in order to prepare for the lesson.

Well if possible I give them a problem to work out beforehand otherwise they always have notes beforehand, they have a breakdown of what’s going to happen. When the maths is quite complicated it really helps if they read beforehand so it helps if they have some sort of idea of some of the new terms.

This was evident in my observation where both the lessons began with the students working out problems that they were given in advance to prepare for the lessons.

The activities that lecturer 1 engaged his students in would depend on the topic that he was teaching. In this case since the topic under discussion was ‘calculus for teaching’ the activities that the students were engaged in are reflected in the activity
sheets. These activities included questions on gradients, derivatives and tangents, all of which are relevant to the teaching of calculus.

It would all depend on the situation. On what I wanted to teach within the topic. For instance if I wanted to teach, for example, the derivative concept via first principles, the student has got no notion of the definition at that stage. So from the average gradient I would now lead on to speaking about the approach of one point to another in the classical way where you arrive at the gradient of the tangent at a particular instance on the graph. So what I’m saying is the particular, the concept that I put across determines the activities that I design so as to gain students’ understanding of the concept.

Lecturer 2 is approaches her teaching in the following manner:

I will, firstly if I am teaching a course I will look at what I’m supposed to teach. I design the course outlines if I’m teaching it for the first time. Then I go through everything. I work through every possible problem in that text book. Then I’ll go find other books and look at how they approach it.

Lecturer 1 allowed students to discover skills and also lectures in the classical style depending on the context of the learning situation.

I would say yes, both skills and conceptual understanding ... and skills yes. Many of these skills are not self discovered, they are taught to students in a lecture style.

This was observed in the video recording. This lecturer always provided clues that could (in his mind) assist students in succeeding in the task. He did not provide them with answers.

Lecturer 2 on the other hand would rather provide clues to solve a problem. This is in keeping with the third element of scaffolding as proposed by Zhao and Orey (as cited in Lipscombe et al. 2008, 5)

If I need them to understand something I try to find a motivating question. It could be a maths question, it doesn’t have to be a concrete activity, that will make them think about the need for what I’m going to introduce. Or I present them this whole big idea to them and show them how this little thing fits.

**PARTICIPANTS’ PERCEPTIONS TO THE LINK BETWEEN CONTENT KNOWLEDGE AND CLASSROOM PRACTICE**

The analysis of the data in the previous sections depicted a strong link between content knowledge of calculus and classroom practice. This was further highlighted by lecturer 1. He elaborated:
To me I feel there is a very strong link between my content knowledge and the way I teach. I am able to emphasize on particular aspects of the content.

Kilpatrick et al. (2001, 372) also argued that the teachers’ content knowledge is important for effective teaching. They argued that the teachers’ content knowledge is important in the development of the students’ proficiency and ability in mathematics. Lecturer 2 also supported this notion:

A deeper knowledge of calculus ... affects how you teach because the deeper your knowledge is, you have a bigger repertoire of examples to draw upon and you can readily come up with counter examples in order to help learners to see conditions when theorems hold and or conditions when rules hold. You are able to, without any problem, come up with relevant examples.

These sentiments expressed by lecturer 2 are supported by Ball and Bass who argued that ‘knowing content is also crucial to being inventive in creating worthwhile opportunities for learning that takes learners’ experiences, interests, and needs into account’ (2000, 86). Even also endorsed these views when she stated that ‘acquiring the basic repertoire gives insights into and a deeper understanding of general and more complicated knowledge’ (1990, 525).

In general both lecturer 1 and lecturer 2 saw a link between content knowledge and classroom practice. Lecturer 1 believed:

Certainly there is ... my classroom practice, the approach that I use is one that can foresee solutions based on the content knowledge, in other words, the classroom practice, the approach that I use is dictated by content knowledge. I have a whole global picture of where I’m going.

Lecturer 2 added a further dimension:

I don’t think you got a one to one relationship, but definitely a deeper content knowledge results in better classroom practice…it is necessary. To have good classroom practice it is necessary to have good content knowledge but not sufficient.

In terms of her classroom practice lecturer 2 was able to respond to students’ queries on several occasions. The one instance was when she explained the link between the gradient of $f(x)$ with $f(x) = g(x)$ in one of the tasks in the activity sheets.

**PARADIGM SHIFT AND CONTRIBUTION TO SUB-DOMAINS OF CONTENT KNOWLEDGE**

Most research on mathematics content knowledge in a South African context has been carried out on practicing school teachers or pre-service teachers. Research on mathematics school teachers content knowledge has been extensively carried out by
many academics like Kazima and Adler (2006), Adler et al. (2002) and Parker and Adler (2005). Research on South African pre-service mathematics teachers has been carried out by many other academics like Brijlall and Maharaj (2011) and Maharajh et al. (2008). Little research (none to the knowledge of the authors) has been carried out on the content knowledge of mathematics university lecturers and their classroom practice in South Africa. This article hence is a shift in research paradigm from the school to the university.

The work by Shulman (1986) was based on teaching and learning on a general setting. Ball et al. (2008) have extended the work of Shulman (1986; 1987) to mathematics specific teaching and learning. They have introduced four sub-domains of content knowledge. Two of these sub-domains viz. common content knowledge and specialized content knowledge are relevant to this study. In fact this study shows that the lecturers practice portrayed features in keeping with the features listed by Ball et al. (2008). In relation to the sub-domain specialized content knowledge the data (during classroom dialogues) is evidence to the following features:

a. Finding an example to make a specific mathematical point.
b. Responding to students’ ‘why’ question.
c. Recognizing what is involved in using a particular representation.
d. Modifying tasks to make learning easier.
e. Evaluation the plausibility of students’ claims.

The lecturers’ design of the worksheet and responses to students’ query employed the knowledge and skills they gained from settings other than teaching. These skills were alluded to during the interviews and evidenced in the lecture profiles provided earlier. This study has therefore provided evidence to justify claims made by Ball et al. (2008). Ball et al. (2008) report on a school context in the United State of America whereas this study reports on a university setting in a South African context.

**CONCLUSION**

What was clear from the findings was that both the lecturers viewed content knowledge as the knowledge that pertains to the understanding of a particular concept or topic in mathematics. This knowledge involves more than knowing the mathematics, it also involves knowing the relationship between various topics and where a particular topic fits into the bigger picture.

The two university lecturers definitely saw a positive correlation between content knowledge and classroom practice. A thorough content knowledge enhanced their classroom practice. It allowed them to assist students by providing clues rather than merely supplying them with answers. This was verified by the actual lecturer-student interactions as viewed in the video recordings. The findings of this study have shown that the depth of content knowledge determined the strength of the activities designed for teaching. Kazima and Adler (2006) state that mathematical knowledge
for teaching involves the restructuring of knowledge to make it accessible to learners. Much more research is therefore needed on a range of classroom contexts. Also researchers have not yet reached consensus on what exactly comprises mathematical knowledge for teaching and this is therefore something that could be pursued further.

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**REFERENCES**


Kazima, M. and J. Adler. 2006. Mathematical knowledge for teaching: Adding to the


