Abstract

This paper assesses the possible fuel consumption savings of combined optimization of the flight profile (altitude and velocity) and wing morphing (wing length, taper, and sweep). A standard business jet in an international standard atmosphere is modeled and dynamic programming is used to solve the resulting optimal control problem.

Keywords: morphing wings; optimal trajectories; dynamic programming

1. INTRODUCTION

Conventional aircraft have fixed wings which are a compromise to be able to perform in different flight conditions. This has the effect that the wings are often not optimal with respect to maneuverability and fuel efficiency. Morphing wings are an interesting option to tackle this problem. In contrast to conventional wings that can only make rather simple shape changes by means of e.g. flaps or slats, morphing wings can make more complex and continuous shape changes (Barbarino et al., 2011). Although the concept of wing morphing is not new, it recently got a lot more attention in research. This is due to new developments in smart materials and environmental issues. According to Seigler et al. (2007), there are numerous applications of morphing technology in the literature, yet there is a notable lack of published work on fundamental approaches to modeling and controlling such aircraft designs. In order for wing morphing to become a viable asset of future aircraft, more research on modeling and control is essential.

With fuel economy in mind, there is another topic that deserves attention: trajectory optimization. Until recently, aircraft flew in narrow pre-determined routes. As the wind has an important influence on the aerodynamics, one could optimize the route (ground track) by taking wind predictions into account. Ascending and descending happens in several steps, also cruising is only allowed at fixed stepped altitudes. Allowing continuous ascents, descents and cruising (flight profile optimization) can save a significant amount of fuel, see e.g. Andreeva-Mori et al. (2011) and Kamgarpour et al. (2011). Furthermore, also the velocity can be optimized (Lovegren and Hansman, 2011).

This paper combines morphing wings and trajectory optimization to minimize fuel consumption. Morphing is taken into account by allowing the length, sweep, and taper ratio of the wing to vary.
Only the vertical trajectory is considered (altitude and velocity), which is called the flight profile in the rest of the paper. Simulations allow for an assessment of the possible fuel savings.

2. AIRCRAFT MODELING

The aircraft is modeled as a 3DOF point mass (Hull, 2007). The equations of motion are:

\[
\begin{align*}
\dot{s} &= v \cdot \cos \gamma, \\
\dot{h} &= v \cdot \sin \gamma, \\
\dot{\psi} &= \frac{1}{m} (T \cos(\alpha + \epsilon_0) - D - mg \cdot \sin \gamma), \\
\dot{\gamma} &= \frac{1}{m} (T \sin(\alpha + \epsilon_0) + L - mg \cdot \cos \gamma), \\
\dot{m} &= -CT,
\end{align*}
\]

with \(s\) (m) the ground distance, \(v\) (m/s) the velocity, \(\gamma\) (rad) the flight path angle, \(h\) (m) the altitude, \(m\) (kg) the aircraft mass, \(T\) (N) the engine thrust, \(\alpha\) (rad) the angle of attack, \(\epsilon_0\) (rad) the thrust angle of attack, \(D\) (N) the drag, \(g = 9.81\) m/s the gravitational acceleration, \(L\) (N) the lift, and \(C\) (kg/N) the thrust specific fuel consumption.

The simulations in this paper are done using a model of a standard business jet (SBJ) flying in an international standard atmosphere, as presented by Hull (2007). The SBJ has two GE CJ610-6 Turbojet engines. To calculate the lift, only the main wing (NACA 64-109) is taken into account. To calculate the friction drag, the whole aircraft is taken into account. The lift and the drag coefficient are a function of the angle of attack \(\alpha\) (rad), the Mach number \(M\) (-), the Reynolds number \(Re\) (-), the wing length \(l\) (m), the wing sweep angle \(\Lambda\) (rad), and the wing taper ratio \(\lambda\) (-). The thrust specific fuel consumption is based on a look-up table.

3. FLIGHT STAGES

A typical flight consists of take-off, climb, cruise, descent, and landing. Each of these stages has different constraints. In this paper, the following is assumed:

• take-off
  o ground run

The aircraft accelerates maximally up to the rotation speed \(v_r = 1.2 v_s\) with \(v_s\) the stall speed at maximum angle of attack. There is wheel friction, with a friction coefficient \(\mu = 0.02\)
The aircraft rotates instantaneously to the maximum angle of attack and takes off. At an altitude of 35 ft the transition ends. During the transition the angle of attack is constant.

During the ground run and transition, the flaps are deployed at 20 deg and the landing gear is out. The engines give maximum thrust. In the simulations the ground effect, flap lift and drag, and landing gear drag are taken into account. It is assumed that the angle of attack can be changed instantaneously. After the transition, the flaps and landing gear are pulled in instantaneously.

- climb

The aircraft climbs to the cruising altitude, using maximum continuous engine thrust when possible. Up to an altitude of 10 000 ft, the calibrated airspeed $v_{ca}$ is limited to 250 knots.

$$v_{ca} = c_0 \sqrt{5 \left( \frac{q}{P_0} \right)^{2/7} - 1},$$

with $c_0$ (m/s) and $P_0$ (Pa) the speed of sound and pressure at sea level, and $q = pv^2/2$ (Pa) the dynamic pressure. The climb angle $\gamma$ is determined based on the maximum climb angle for a quasi-steady climb:

$$\gamma = \min \left( K \frac{T - D}{L}, \gamma_{max} \right),$$

with $K$ (-) the climb factor and $\gamma_{max} = 8$ deg for passenger comfort.

- cruise

The cruise is defined by a fixed Mach number and altitude. If the cruise Mach number is not yet reached at the end of the climb, the aircraft will keep accelerating at maximum continuous thrust.

4. OPTIMAL FLIGHT PROBLEM AND SOLUTION METHOD

The goal is to minimize the fuel consumption by optimizing the flight profile (altitude and velocity), and by allowing the wings to morph. The optimal flight problem is thus to minimize the time integral of the fuel mass flow rate $m_f$ (kg/s):

$$\min_{x, T, \mathcal{A}, \mathcal{A}} \int_0^T m_f dt,$$

subject to the model equations (1).

Because of the model complexity, non-linearities, and the look-up table for the thrust specific fuel consumption, dynamic programming is used to solve the
optimal flight problem. Dynamic programming (Bellman, 1957) is a powerful method to solve complex optimal control problems, but unfortunately requires long calculation times and a rather rough discretization of the state variables.

The optimal flight problem is discretized in the state space of the distance $s$, altitude $h$ and Mach number $M$ (The Mach number is used instead of the velocity.) The morph controls are also discretized. The control problem is converted to a problem with $K$. The base profile is determined by optimizing the aforementioned characteristic variables with a Nelder-Mead simplex method. To further decrease the calculation time, only one wing shape variable is allowed to vary.

5. **SIMULATIONS**

The following reference flight is used to assess possible fuel savings:

- total distance: $s = 300$ km,
- cruise altitude: $h_{\text{cruise}} = 30000$ ft,
- cruise Mach number: $M_{\text{cruise}} = 0.8$,
- climb factor: $K = 0.75$,
- limited climb angle: $\gamma \leq 8$ deg,
- below 10 000 ft: $v_{\text{cas}} \leq 250$ knots.

Five optimized flights are compared to the reference flight:

1. optimal profile: the flight profile is optimized while keeping the wings fixed,
2. optimal wing length: the wing length is optimized while flying the reference flight,
3. optimal profile and wing length: the flight profile and wing length are optimized simultaneously,
4. optimal wing taper: the wing taper is optimized while flying the reference flight,
5. optimal profile and wing taper: the flight profile and wing taper are optimized simultaneously.

Simulation results for an optimized wing sweep are not given, as this hardly results in fuel savings. The following constraints are applied: $2.6 < l < 8$ m and $0.25 < \lambda < 1$.

Figures 1–4 show the simulation results for the different flights. Table 1 gives the possible fuel savings. For this specific simulation example, optimizing the profile, Mach number, and wing taper results in the lowest fuel consumption.
Table 1. Fuel savings

<table>
<thead>
<tr>
<th></th>
<th>$m_f$ (kg)</th>
<th>(%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>reference</td>
<td>418.5</td>
<td>100.0</td>
</tr>
<tr>
<td>optimal profile</td>
<td>404.8</td>
<td>96.7</td>
</tr>
<tr>
<td>optimal length</td>
<td>416.3</td>
<td>99.5</td>
</tr>
<tr>
<td>optimal length and profile</td>
<td>393.4</td>
<td>94.3</td>
</tr>
<tr>
<td>optimal taper</td>
<td>404.7</td>
<td>96.7</td>
</tr>
<tr>
<td>optimal taper and profile</td>
<td>381.0</td>
<td>91.1</td>
</tr>
</tbody>
</table>

Fig. 1. comparison of altitude

Fig. 2. comparison of Mach number
6. CONCLUSIONS

This paper assesses the possible fuel consumption savings of combined optimization of the flight profile (altitude and velocity) and wing morphing (wing length, taper, and sweep). A standard business jet in an international standard atmosphere is modeled and dynamic programming is used to solve the resulting optimal control problem. To speed up the calculations, the dynamic programming algorithm only searches in the neighborhood of a base profile. This profile is characterized by a cruising altitude and Mach number, and a climb factor.
For a take-off, climb and short cruise, combined optimization of the flight profile and wing taper can save almost 10% of fuel. Because of the limited validity of this one specific example, further research is needed, but it seems that wing morphing can be an interesting means to lower the fuel consumption in civil aviation.

7. REFERENCES


